Superconductivity in Strongly-Coupled Amorphous and Nanostructured Materials

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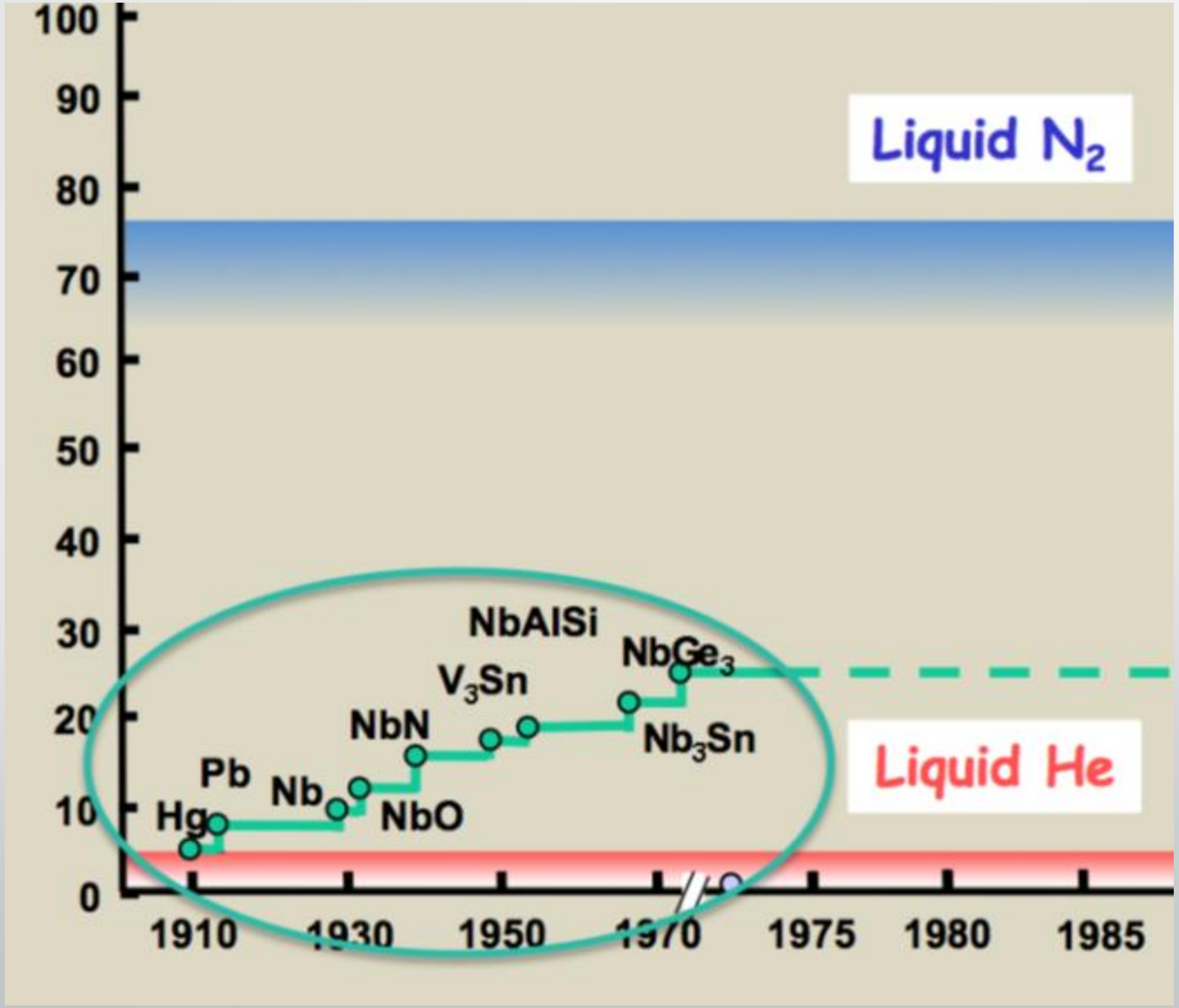
Superconducting Elements and Alloys with $T_c \leq 25$ K

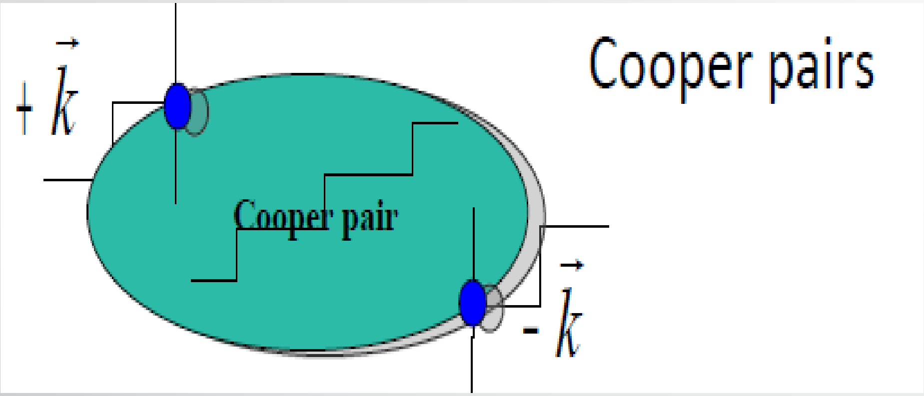


BCS Theory

BCS: theory for conventional superconductivity (1957)

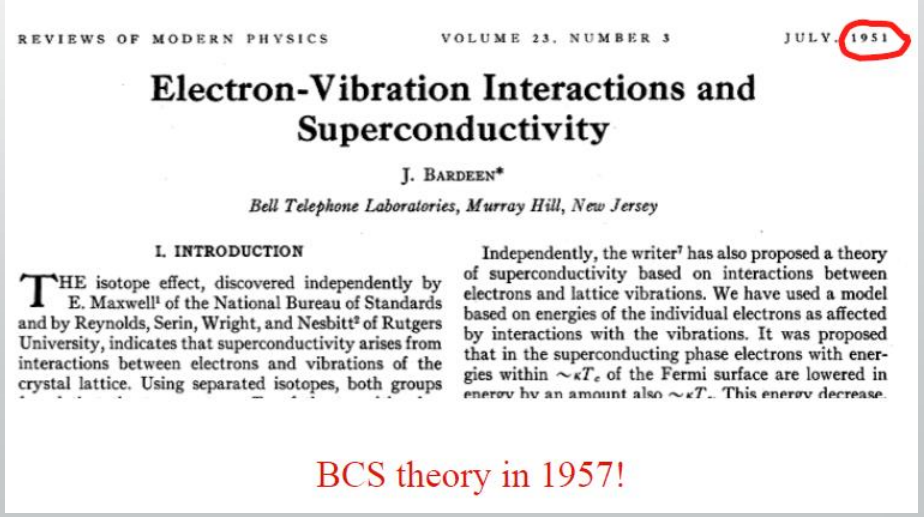
Nobel prize (1972)

J. Bardeen, L. Cooper, R. Schrieffer



Cooper problem: **The Fermi liquid is unstable towards any arbitrarily small attractive *e-e* interaction**

Cooper pairs form zero-spin (singlet) bosons transporting electric current without dissipation



REVIEWS OF MODERN PHYSICS VOLUME 23, NUMBER 3 JULY, 1951

Electron-Vibration Interactions and Superconductivity

J. Bardeen*

Bell Telephone Laboratories, Murray Hill, New Jersey

I. INTRODUCTION

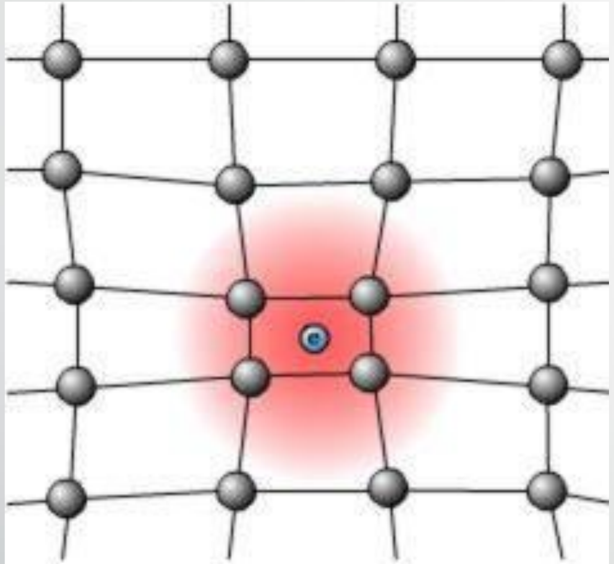
THE isotope effect, discovered independently by E. Maxwell[1] of the National Bureau of Standards and by Reynolds, Serin, Wright, and Nesbitt[2] of Rutgers University, indicates that superconductivity arises from interactions between electrons and vibrations of the crystal lattice. Using separated isotopes, both groups

Independently, the writer[7] has also proposed a theory of superconductivity based on interactions between electrons and lattice vibrations. We have used a model based on energies of the individual electrons as affected by interactions with the vibrations. It was proposed that in the superconducting phase electrons with energies within $\sim\kappa T_c$ of the Fermi surface are lowered in energy by an amount also $\sim\kappa T_c$. This energy decrease

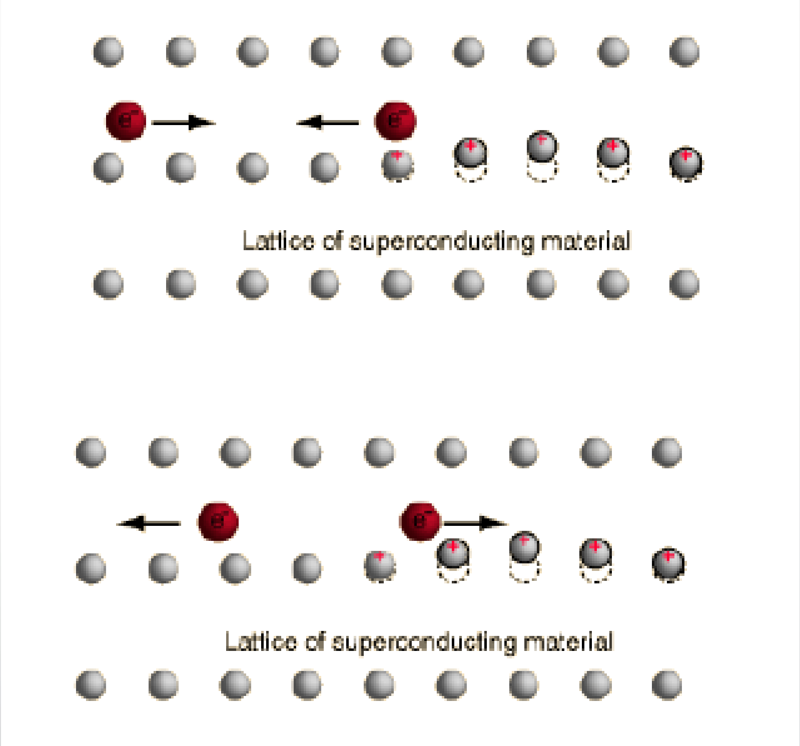
BCS theory in 1957!

BCS Theory

How can phonons produce an attraction among electrons?



Lattice deformation



The lattice deformation caused by an electron attracts the second one.

BCS Theory

BCS Hamiltonian:

$$\hat{H} = \sum_{k\sigma} \varepsilon_k \hat{n}_{k\sigma} + \sum_{k,k'} V_{kk'} b_{k'}^{\dagger} b_k$$

Kinetic energy

Scattering of pairs from k to k'

All pairs have the same phase (coherence)

$V_{kk'}$ pairing potential arises from a competition of the phonon attraction and Coulomb repulsion

BCS gap equation:

$$\Delta_k = -\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2\sqrt{\xi_{k'}^2 + \Delta_{k'}^2}} \tanh \frac{\sqrt{\xi_{k'}^2 + \Delta_{k'}^2}}{2k_B T} \rightarrow 1 - f(E_k) - f(E_{k'})$$

T_c *is* the maximum temperature at which a nontrivial solution is possible

$$T_c = 1.14\,\Theta\, e^{-1/\lambda}$$

BCS Theory

$$T_c = 1.14\,\Theta\, e^{-1/\lambda}$$

$$\lambda = N(E_F) \langle V_{kk'} \rangle_{E_F}$$

It is a weak coupling theory ($\lambda \ll 1$)

Interaction potential $V=V_{(ph)}+V_{(el)}$ includes:

•Phonon-mediated attraction

•Direct electron-electron repulsion

$$\boxed{2\Delta_0/T_c = 3.52}$$

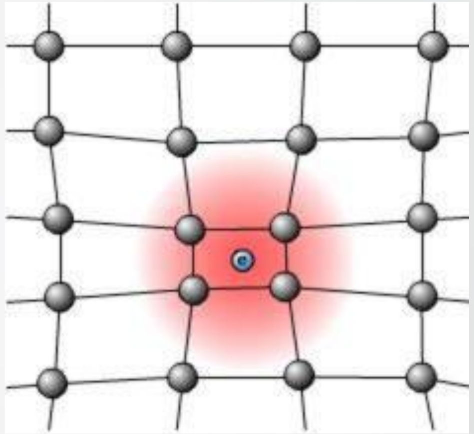
To get superconductivity, David needs to defeat Goliath

Phonon-mediated attraction, phonon frequencies 10 - 100 meV

Screened Coulomb potential, plasma frequency, few eV

Migdal-Eliashberg Theory

The Eliashberg theory of superconductivity is based on a dynamical electron-phonon interaction as opposed to a static interaction present in BCS theory.



Basic assumption: $\Theta_D/E_F \ll 1$

Equation to determine the renormalization function

$$|g_{\mathbf{pp'}}|^2\delta(\omega-\Omega_{\mathbf{p}-\mathbf{p'}}) \Longrightarrow \frac{1}{N(0)}\sum_{\mathbf{p}}\frac{1}{N(0)}\sum_{\mathbf{p'}}|g_{\mathbf{pp'}}|^2 \times$$
$$\times\, \delta(\omega-\Omega_{\mathbf{p}-\mathbf{p'}})\delta(\varepsilon_{\mathbf{p}})\delta(\varepsilon_{\mathbf{p'}}) \equiv \frac{1}{N(0)}\alpha^2(\omega)F(\omega), \quad (4)$$

Phonon density of states weighted by the coupling with electrons at E_F

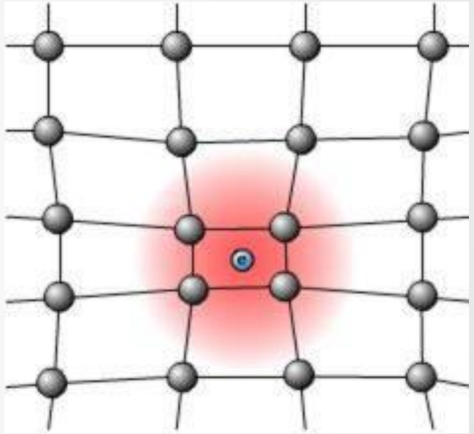
$$F(\omega) = \sum_{\mathbf{q}}\delta(\omega-\Omega_{\mathbf{q}})$$

Phonon density of states

Migdal-Eliashberg Theory

Basic assumption: $\Theta_D/E_F \ll 1$

Electron-phonon coupling constant



$$\lambda = 2\int_0^\infty \frac{d\omega}{\omega}\alpha^2(\omega)F(\omega)$$

$$m^\star = m(1+\lambda)$$

$$\lambda = \frac{2}{N(0)}\int \frac{d\omega}{\omega}\sum_{\mathbf{p}}\sum_{\mathbf{p}'}|g_{\mathbf{p}\mathbf{p}'}|^2\delta(\omega-\Omega_{\mathbf{p}-\mathbf{p}'})\times$$
$$\times\,\delta(\varepsilon_{\mathbf{p}})\delta(\varepsilon_{\mathbf{p}'}-\Omega_{\mathbf{p}-\mathbf{p}'}),$$

Eliashberg Theory

Basic assumption: $\Theta_D/E_F \ll 1$

Eliashberg theory is a theory of superconductivity that describes the role of phonons in providing the attractive interaction between two electrons.

Separation between phonon-exchange & Coulomb parts

Eliashberg equations for a superconductor

$$[1 - Z(\varepsilon)]\varepsilon = -\int_{-D}^{D} d\varepsilon' K(\varepsilon', \varepsilon) \times \\ \times \operatorname{Re} \frac{\varepsilon'}{\sqrt{\varepsilon'^2 - \Delta^2(\varepsilon')}} \operatorname{sign} \varepsilon,$$

$$Z(\varepsilon)\Delta(\varepsilon) = \int_{-D}^{D} K(\varepsilon', \varepsilon) \times \\ \times \operatorname{Re} \frac{\Delta(\varepsilon')}{\sqrt{\varepsilon'^2 - \Delta^2(\varepsilon')}} \operatorname{sign} \varepsilon,$$

$$K(\varepsilon', \varepsilon) = \frac{1}{2} \int_0^\infty d\omega \alpha^2(\omega) F(\omega) \times \\ \times \left\{ \frac{\operatorname{th} \frac{\varepsilon'}{2T} + \operatorname{cth} \frac{\omega}{2T}}{\varepsilon' + \omega - \varepsilon - i\delta} - \frac{\operatorname{th} \frac{\varepsilon'}{2T} - \operatorname{cth} \frac{\omega}{2T}}{\varepsilon' - \omega - \varepsilon - i\delta} \right\}$$

Eliashberg Theory

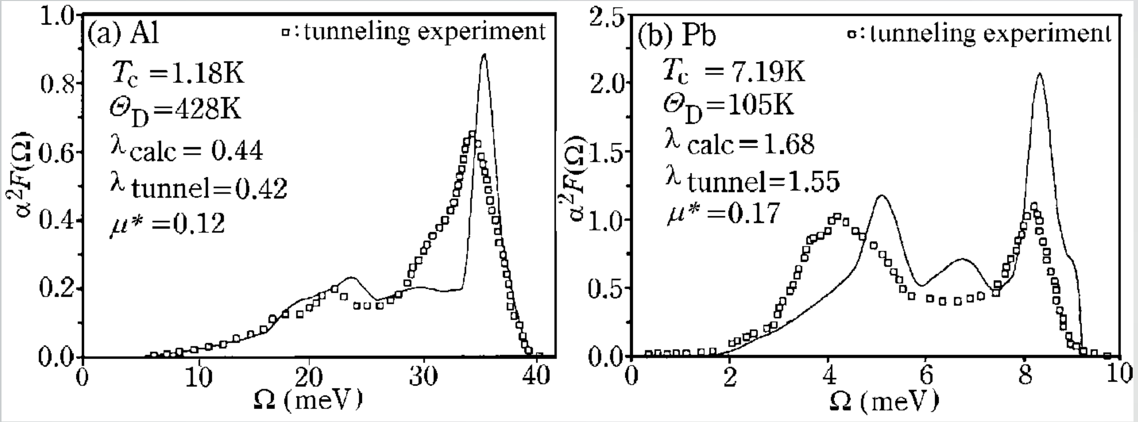
Coulomb pseudopotential

$$\boxed{\mu^* = \frac{\mu_c}{1+\mu_c \ln(E_F/\omega_c)}} \quad \boxed{\mu_c = N(0)\left\langle \frac{V_c(q)}{1+V_c(q)\Pi(\mathbf{q},0)} \right\rangle}$$

$V_c(q) \propto q^{-2}$、 $\Pi(\mathbf{0},0) = 2N(0) \rightarrow \mu_c \approx N(0)/\Pi(\mathbf{0},0) \approx 0.5$

$E_F/\omega_c = 0.01 - 0.001 \rightarrow \mu^* = 0.11 - 0.15$

Eliashberg Theory



$$\lambda = 2\int_0^{\infty} \frac{d\omega}{\omega}\alpha^2(\omega)F(\omega)$$

McMillan equation

$$T_c = \frac{T_D}{1{,}45}\exp\left\{-\frac{1{,}04(1+\lambda)}{\lambda-\mu^*(1+0{,}62\lambda)}\right\}$$

BCS equation

$$T_c = 1.14\,\Theta\, e^{-1/\lambda}$$

Anderson’s theorem

In the field of superconductivity, **Anderson's theorem** states that superconductivity in a conventional superconductor is robust with respect to (non-magnetic) disorder in the host material.

One consequence of Anderson's theorem is that the critical temperature T_c of a conventional superconductor barely depends on material purity, or more generally on defects. This concept breaks down in the case of very strong disorder, e.g. close to a superconductor-insulator transition. Also, it does not apply to unconventional superconductors. In fact, strong suppression of T_c with increasing defect scattering, thus non-validity of Anderson's theorem, is taken as a strong indication for superconductivity being unconventional.

Wikipedia

Amorphous Superconductors

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Effective theory of superconductivity in strongly coupled amorphous materials

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Typically, in amorphous materials

$$\lambda > 1$$

$$\Delta \gg 1.76 k_B T_c \equiv \Delta_{BCS}$$

Early work established that structural atomic disorder may promote an increase of λ upon going from the crystalline to the amorphous state.

Ga ↑, Bi ↑, Be ↑, Pb ↓

… lack of theoretical frameworks able to disentangle and describe, in a reductionist way, the effect of structural disorder on the strong-coupling superconductivity. This delay of theory is imputable to the difficulty of providing an effective description of structural disorder on the vibrational spectrum of amorphous solids, where a long-standing issue is represented by the so-called boson peak or excess of soft vibrational modes which shows up in the vibrational density of states upon normalizing it by the Debye law ω^2 and for which a deeper understanding has emerged only recently as follows.

Amorphous Superconductors

Phonon Green's function

$$\frac{\partial^2 u_\lambda}{\partial t^2} = v_\lambda^2 \Delta u_\lambda + D_\lambda \frac{\partial \Delta u_\lambda}{\partial t}$$

$$G_\lambda(\omega, k) = \frac{1}{\omega^2 - \Omega_\lambda^2(k) + i\,\omega\,\Gamma_\lambda(k)}$$

$$\Gamma_\lambda(k) = D_\lambda\, k^2$$

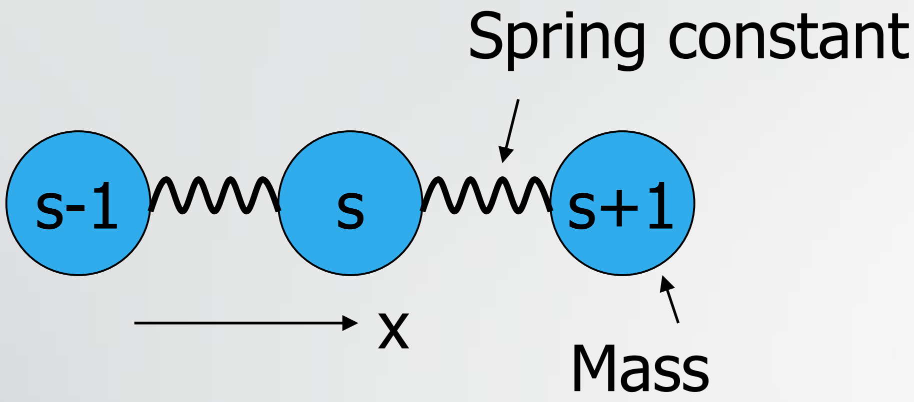
$\lambda = L$ or $\lambda = T$

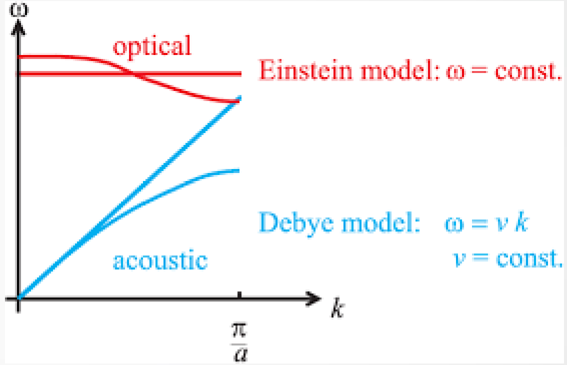
Phonon spectral function

$$\mathcal{B}_\lambda(\omega, k) = -\frac{1}{\pi}\,\mathrm{Im}G_\lambda(k, \omega + i\delta)$$

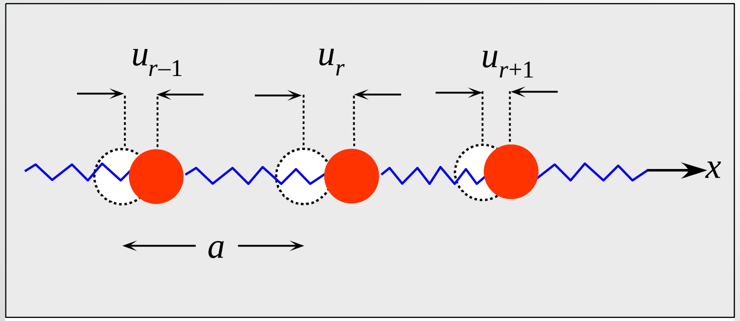
$$\mathcal{B}(\omega, k) = \frac{\omega\,\Gamma(k)}{\pi\{[\omega^2 - \Omega^2(k)]^2 + \omega^2\,\Gamma^2(k)\}}$$

Eliashberg Theory

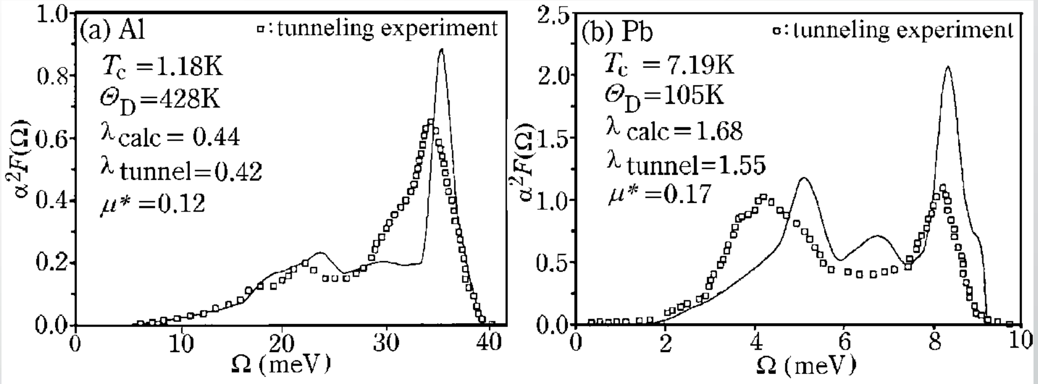




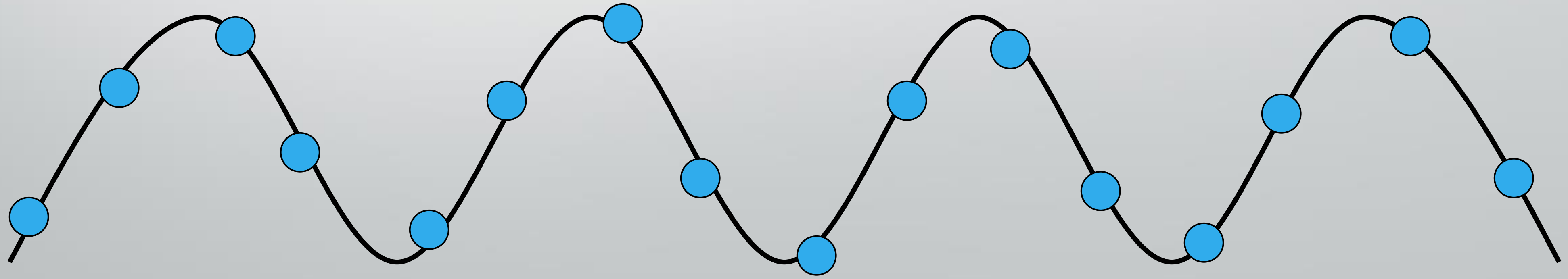
$\lambda = L$



Atoms executing longitudinal vibrations parallel to x.



$\lambda = T$



Amorphous Superconductors

Introduction of the Eliashberg function

$$\alpha^2 F(\omega) = \frac{1}{\mathcal{N}(\mu)^2} \sum_{\vec{k},\vec{k}'} \alpha^2 F(\vec{k},\vec{k}',\omega)\,\delta(\epsilon_{\vec{k}} - \mu)\,\delta(\epsilon_{\vec{k}'} - \mu)$$

$$\alpha^2 F(\vec{k},\vec{k}',\omega) \equiv \mathcal{N}(\mu)\,|g_{\vec{k},\vec{k}'}|^2\,\mathcal{B}(\vec{k}-\vec{k}',\omega)$$

$$\alpha^2 F(\omega) = \frac{g^2}{4\,(2\pi)^4\,N} \int \sum_{\lambda} \mathcal{B}_\lambda(X^2,\omega)\,d\phi_k\,d\phi_{k'}$$

$$\mathcal{B}(X^2,\omega) = \frac{\omega\,D\,X^2}{\pi\,(\omega^2 - v^2 X^2)^2 + \omega^2 D^2 X^4}$$

$$X^2 \equiv 2\mu\,(1 - \cos(\phi_k - \phi_{k'}))$$

Amorphous Superconductors

three separate regimes and corresponding excitations

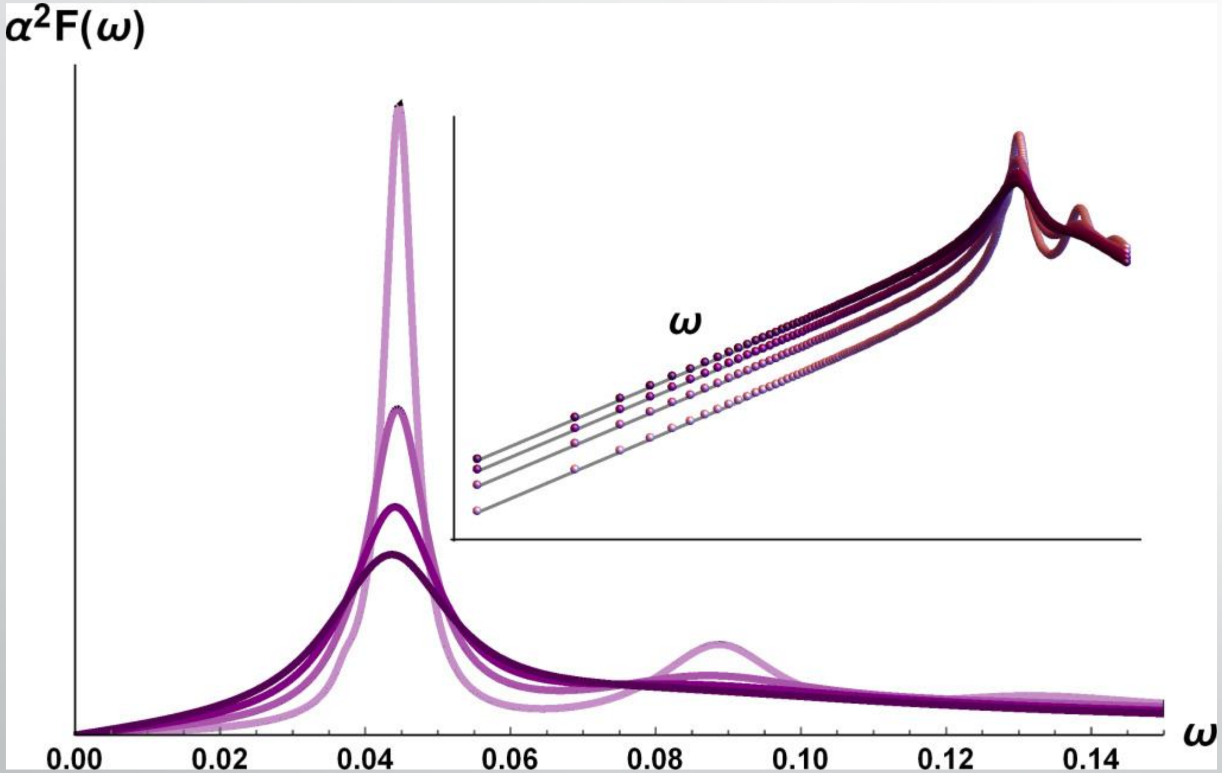
(i) *Propagons*. This is the limit in which dissipation is not dominant, $vk \gg Dk^2$, and the excitations are still well-defined quasiparticles undergoing ballistic motion.

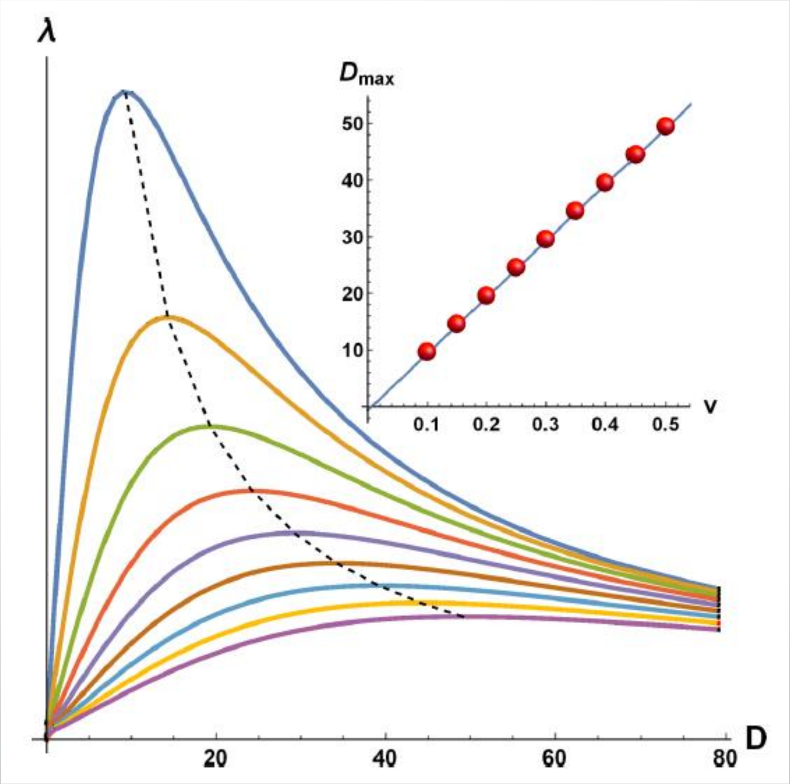
(ii) *Diffusons*. This regime appears beyond the Ioffe-Regel limit $vk \sim Dk^2$ at which the dissipative term becomes dominant. At this point, there are not well-defined quasiparticles anymore, and the dynamics is totally incoherent, collective, and diffusive. In this range, it does not make sense to think of ballistic concepts such as the mean free path of propagation, simply because there are no propagating particles.

(iii) *Locons*. This is the extreme limit, usually relevant close to the edge of mobility near the Debye frequency, in which the modes are completely (Anderson) localized. Not only do the modes not propagate, but also their diffusion constant vanishes.

Amorphous Superconductors

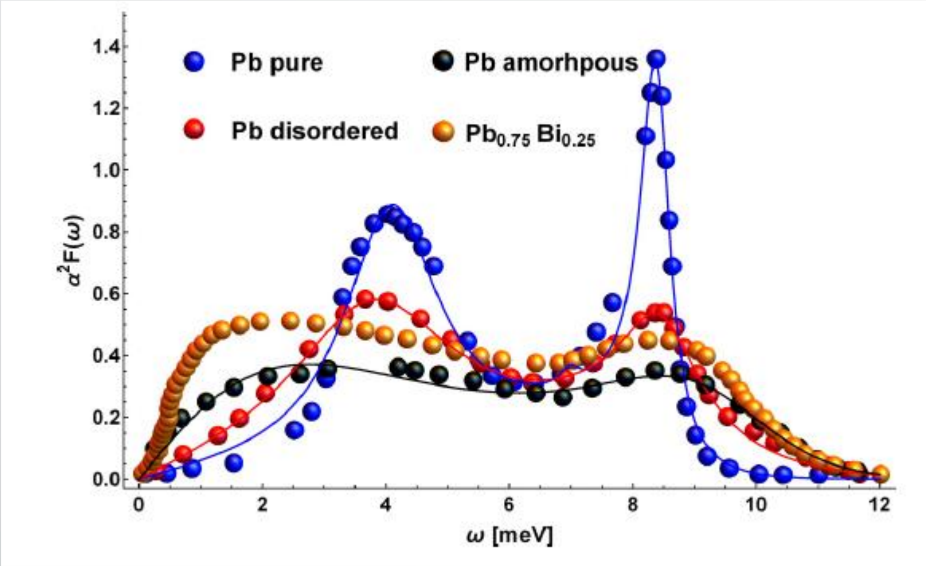
we present theoretical predictions from the above model which show the linear in frequency behavior of the Eliashberg function $\alpha^2F(\omega)$ in the low-ω limit





Amorphous Superconductors

The Lead Puzzle



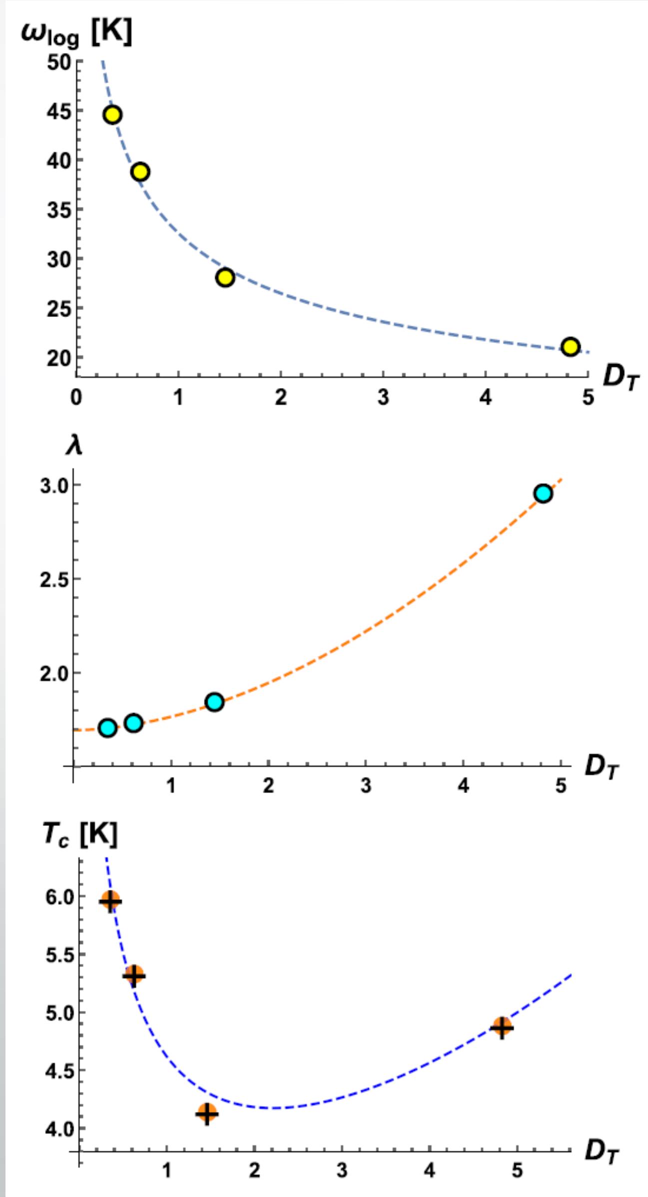
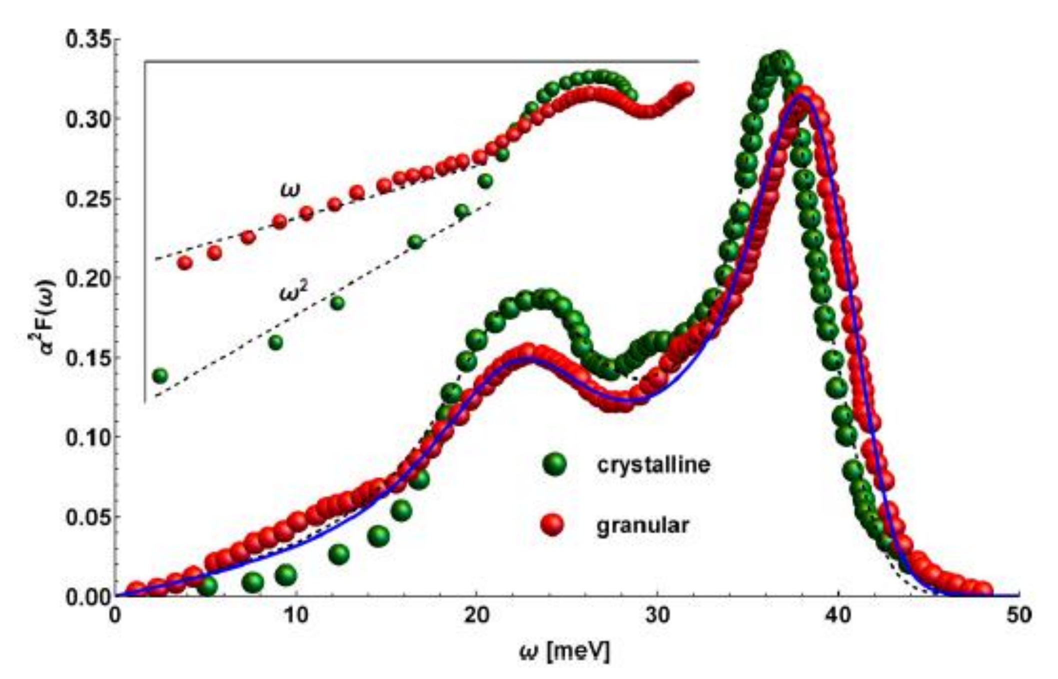


TABLE I. Data of vibrational excitation diffusivities obtained from the fit in Fig. 3 using the theoretical model, Eq. (8).

	Pb-based materials			
	Pure	Granular	$Pb_{0.9}Cu_{0.1}$	$Pb_{0.75}Bi_{0.25}$
D_T	0.358	0.582	1.735	4.90057
D_L	0.116	0.348	0.366	0.55303

Amorphous Superconductors

The Aluminum Puzzle



	Aluminum	
	Crystalline	Granular
D_T (fit)	2.2	2.4
D_L (fit)	1.15	1.6
λ (interpolation)	0.358	0.410
λ (fit)	0.376	0.430
$\omega_{\log}$ (interpolation)	25.2 meV	20.6 meV
$\omega_{\log}$ (fit)	23.2 meV	20.4 meV
T_c (interpolation)	0.60 K	1.71 K
T_c (fit)	0.76 K	2.11 K

Nanostructured Superconductors

SCIENTIFIC REPORTS
natureresearch

The influence of phonon softening on the superconducting critical temperature of Sn nanostructures

Kelly Houben[1], Johanna K. Jochum[1*], Sebastien Couet[2], Enric Menéndez[2], Thomas Picot[1], Michael Y. Hu[3], Jiyong Y. Zhao[3], E. Ercan Alp[3], André Vantomme[2], Kristiaan Temst[2] & Margriet J. Van Bael[1]

Sample	Type	Nominal thickness (nm)	Average island height (nm)	Average β-Sn crystallite size (nm)
isl60	nano islands	60	100 ± 21	68 ± 9
isl40	nano islands	40	68 ± 17	54 ± 6
clus46	cluster film	46	—	123 ± 14

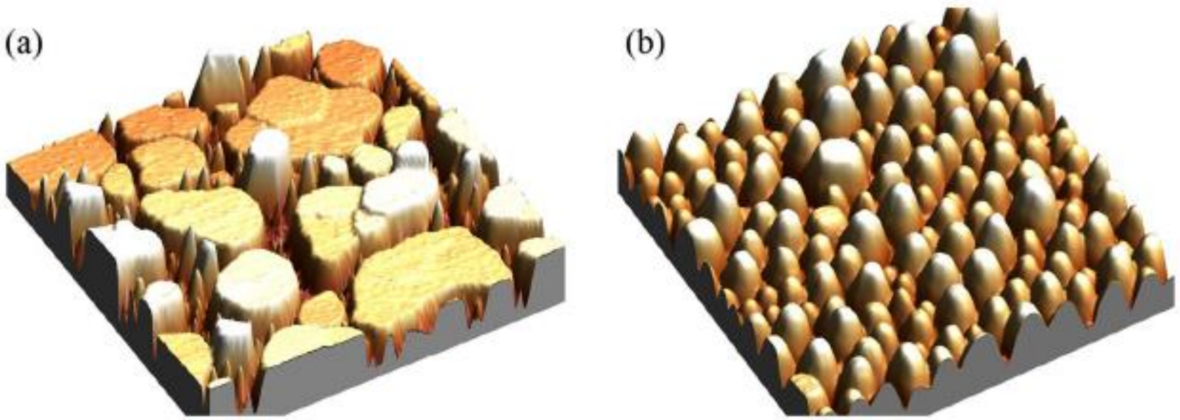
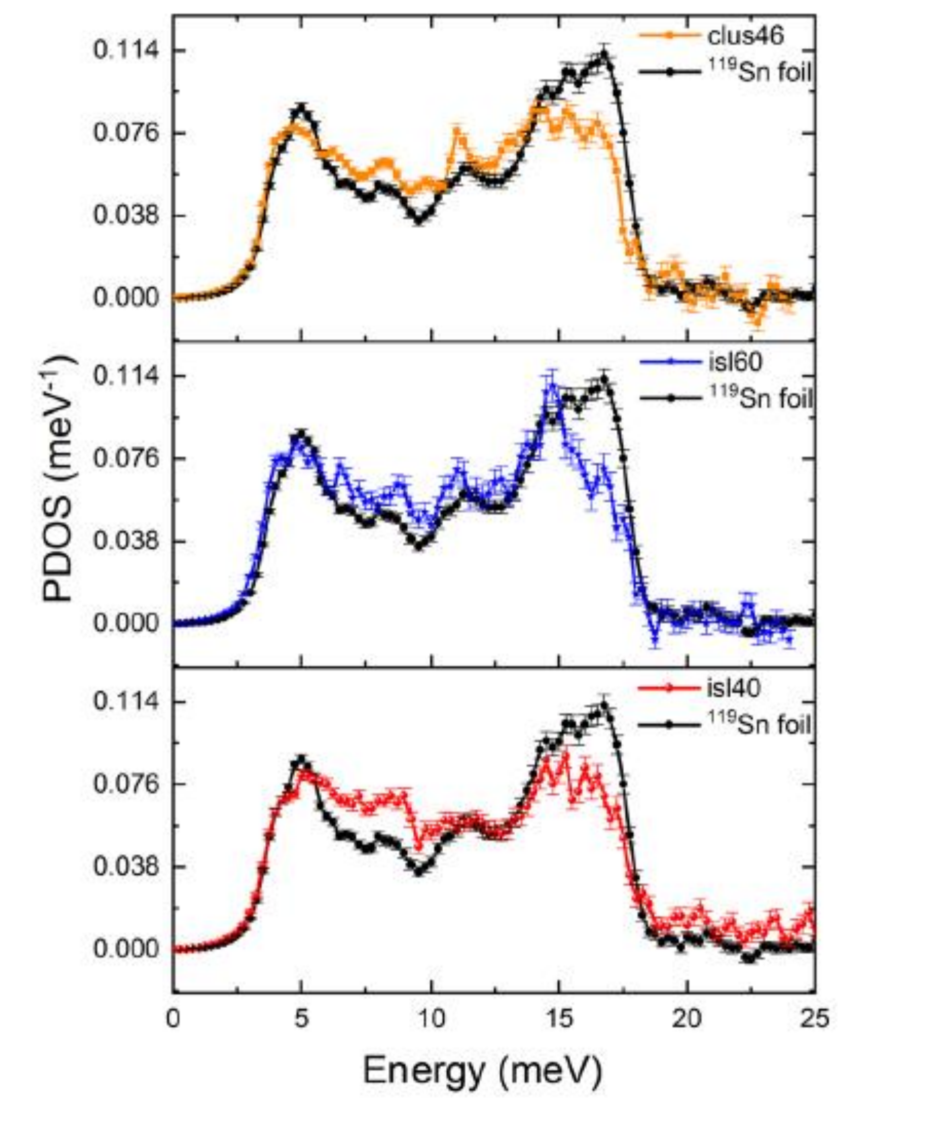


Figure 1. (a) AFM image of Sn islands (height scale = 139 nm, nominal thickness = 40 nm, image size = 7 μm × 7 μm); (b) AFM image of Sn cluster-assembled film, (height scale = 53 nm, thickness = 50 nm, image size = 1 μm × 1 μm).

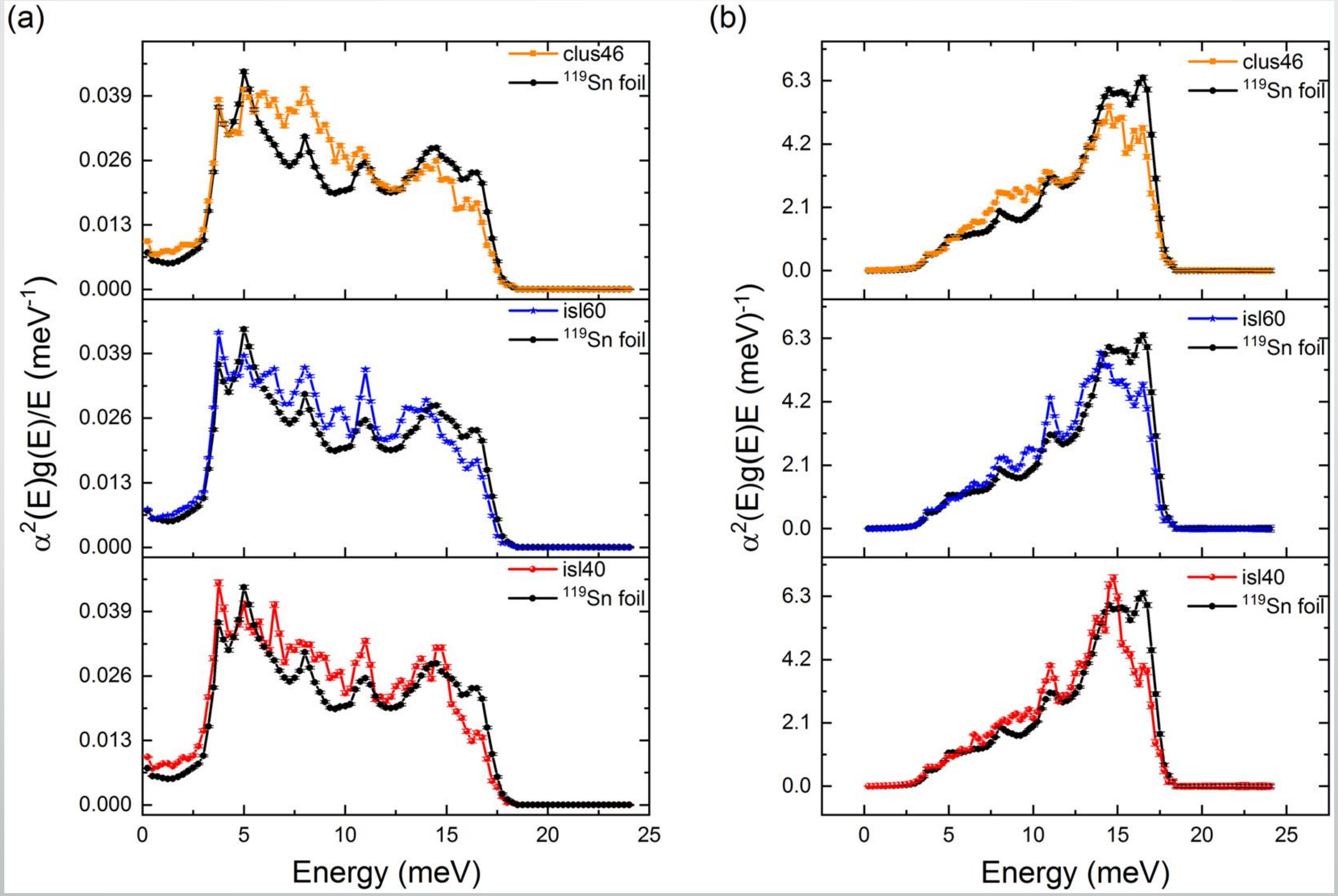
Nanostructured Superconductors

Sample	T_C (K)	H_{C2} (mT)	$\xi(0)$ (nm)	l (nm)
^{119}Sn foil	3.85 ± 0.05	26 ± 6	113 ± 13	75 ± 17
isl60	4.17 ± 0.05	35 ± 8	97 ± 11	56 ± 12
isl40	4.38 ± 0.05	41 ± 6	90 ± 7	48 ± 7
clus46	3.93 ± 0.05	61 ± 5	74 ± 3	32 ± 3



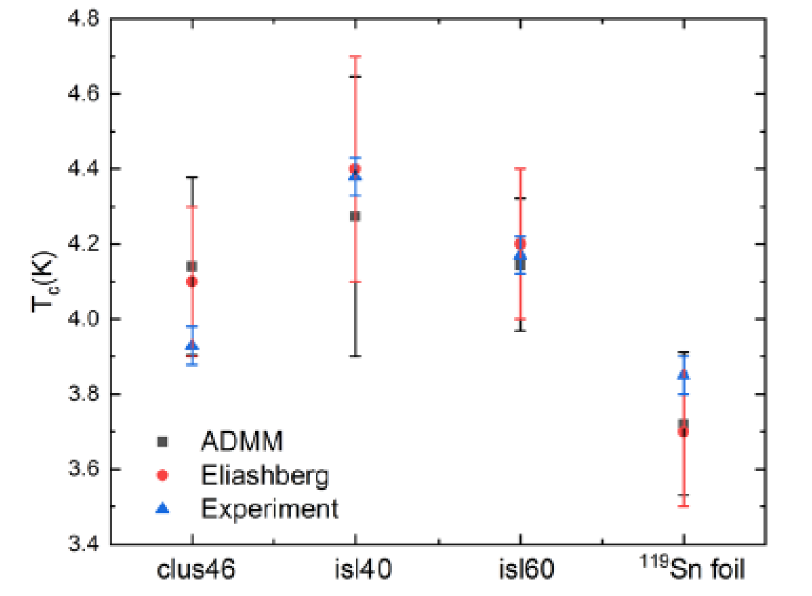
We have studied the PDOS of different Sn nanostructures by nuclear resonant inelastic X-ray scattering and compared them to the PDOS of a bulk Sn foil. We found a decrease of high-energy phonon modes, a small enhancement of low-energy phonon modes and a general broadening of the PDOS features/

Nanostructured Superconductors



Nanostructured Superconductors

Sample	λ	$\langle\Omega^2\rangle^{1/2}$ (meV)	$\int g(E)EdE$ (meV)	ω_{ln} (meV)	$T_{C,cal}^{ADMM}$ (K)	$T_{C,cal}^{E}$ (K)	$T_{C,exp}$* (K)
^{119}Sn foil	0.76 ± 0.04	10.4 ± 0.2	11.62 ± 0.03	8.1 ± 0.2	3.7 ± 0.2	3.7 ± 0.2	3.85
isl60	0.82 ± 0.03	9.9 ± 0.3	10.82 ± 0.04	7.8 ± 0.2	4.1 ± 0.2	4.2 ± 0.2	4.17
isl40	0.84 ± 0.06	9.8 ± 0.4	10.74 ± 0.05	7.6 ± 0.3	4.3 ± 0.4	4.4 ± 0.3	4.38
clus46	0.82 ± 0.04	9.8 ± 0.3	10.36 ± 0.03	7.6 ± 0.2	4.1 ± 0.2	4.1 ± 0.2	3.93



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