

Theoretical Quantum Optics

Quantum Correlation Measurements

Werner Vogel

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Introduction

General quantum correlations of light

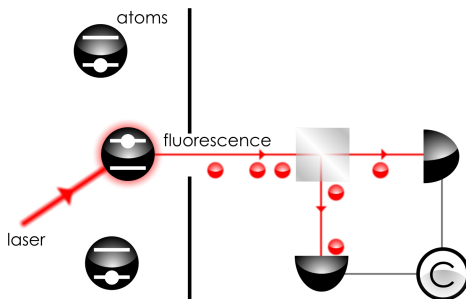
Homodyne correlation measurements

Click counting measurements

Summary

Photon antibunching¹

- First demonstration of nonclassical light: photon antibunching



¹H.J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).

Photon antibunching

- Violation of Schwarz inequality: $\langle \mathcal{T} : \hat{I}(0) \hat{I}(\tau) : \rangle > \langle : [\hat{I}(0)]^2 : \rangle$
- ⇒ Based on normal-and time-ordered correlation functions!

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Photon Antibunching in Resonance Fluorescence

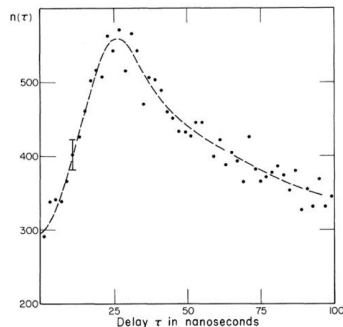
H. J. Kimble,^(a) M. Dagenais, and L. Mandel

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

(Received 22 July 1977)

The phenomenon of antibunching of photoelectric counts has been observed in resonance fluorescence experiments in which sodium atoms are continuously excited by a dye-laser beam. It is pointed out that, unlike photoelectric bunching, which can be given a semi-classical interpretation, antibunching is understandable only in terms of a quantized electromagnetic field. The measurement also provides rather direct evidence for an atom undergoing a quantum jump.

$$\langle n(\tau) \rangle = N_s \Delta \tau \alpha_2 \langle \mathcal{T} : \hat{I}_1(t) \hat{I}_2(t + \tau) : \rangle / \langle \hat{I}_1(t) \rangle, \quad (6)$$



Leonard Mandel visiting Rostock



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Definitions of quantum correlations

- An N -mode state of light, $\hat{\rho}_{\text{cl}}$, is called classical if it can be written as²

$$\hat{\rho}_{\text{cl}} = \int dP(\alpha) |\alpha\rangle \langle \alpha|, \text{ with } |\alpha\rangle = |\alpha_1\rangle \otimes \dots \otimes |\alpha_N\rangle \text{ and } P \equiv P_{\text{cl}} \geq 0$$

- An N -partite state $\hat{\sigma}$ is called separable if it can be written as³

$$\hat{\sigma} = \int dP(\mathbf{a}) |\mathbf{a}\rangle \langle \mathbf{a}|, \text{ with } |\mathbf{a}\rangle = |a_1\rangle \otimes \dots \otimes |a_N\rangle \text{ and } P \equiv P_{\text{cl}} \geq 0$$

- A state $\hat{\rho}$ is nonclassical [entangled] if $\hat{\rho} \neq \hat{\rho}_{\text{cl}}$ [$\hat{\sigma}$]
- General relation: entanglement \Rightarrow quantum correlation

\Rightarrow Entangled states are a subset of quantum correlated ones!

- Applications in quantum technologies⁴

²U. M. Titulaer and R. J. Glauber, Phys. Rev. **140**, B676 (1965).

³R. F. Werner, Phys. Rev. A **40**, 4277 (1989).

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General quantum correlations of light⁵

- Glauber/Sudarshan function $P(\alpha) = P(\alpha_1, \dots, \alpha_N) \Rightarrow P$ functional:

$$P(\{E^{(+)}(i)\}) = \left\langle \mathcal{T} : \prod_{i=1}^k \hat{\delta}(\hat{E}^{(+)}(i) - E^{(+)}(i)) : \right\rangle, \quad i \equiv (\mathbf{r}_i, t_i)$$

- Nonclassical correlations: $P \not\geq 0$

\Rightarrow Hierarchy of conditions for field correlation functions, such as:

$$\langle \mathcal{T} : \Delta \hat{E}(1) \Delta \hat{I}(2) : \rangle^2 > \langle : [\Delta \hat{E}(1)]^2 : \rangle \langle : [\Delta \hat{I}(2)]^2 : \rangle$$

- Correlation functions of higher orders are accessible!

\Rightarrow Detection: homodyne correlation measurements

⁵W. Vogel, Phys. Rev. Lett. **100**, 013605 (2008).

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Squeezing and Anomalous Moments in Resonance Fluorescence

Werner Vogel

Institut für Physik, Hochschule Güstrow, Goldberger Strasse 12, O-2600 Güstrow, Germany

(Received 5 June 1991)

A scheme for measuring squeezing in resonance fluorescence from a single trapped and cooled ion is proposed, which is based on the observation of photon pair correlations after beating the fluorescence with a local oscillator. In addition to squeezing, anomalous moments and sub-Poissonian photon statistics of the fluorescence contribute to the nonclassical behavior of the light in homodyne detection.

PACS numbers: 42.50.Dv, 03.65.-w, 32.80.-t, 42.50.Kb

- More details of the theory⁶
- ⇒ Applies in case of strong losses, e.g., in resonance fluorescence
 - Related technique: conditioned homodyne detection⁷
- ⇒ Applied in cavity QED⁷ and trapped ion experiments⁸

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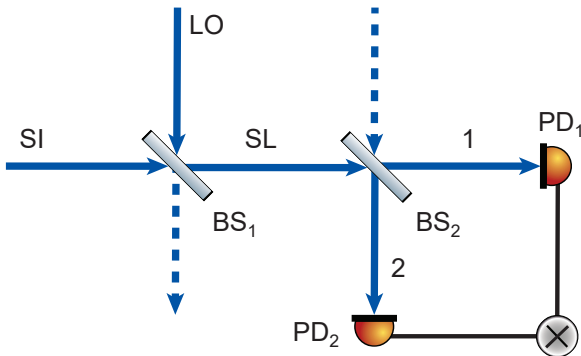
⁶W. Vogel, Phys. Rev. A **51**, 4160 (1995).

⁷H. J. Carmichael, H. M. Castro-Beltran, G. T. Foster, L. A. Orozco, PRL **85**, 1855 (2000).

⁸S. Gerber, D. Rotter, L. Slodicka, J. Eschner, H. J. Carmichael, and R. Blatt, PRL **102**, 183601 (2009).

Homodyne intensity correlation measurement⁹

- The measurement setup:



⁹W. Vogel, Phys. Rev. Lett. **67**, 2450 (1991).

Homodyne intensity correlation measurement⁹

- Accessible intensity correlation functions:

$$\Delta G^{(2,2)} = \left\{ \left\langle [\hat{E}^{(-)}(\mathbf{r}, t)]^2 [\hat{E}^{(+)}(\mathbf{r}, t)]^2 \right\rangle - \left\langle \hat{E}^{(-)}(\mathbf{r}, t) \hat{E}^{(+)}(\mathbf{r}, t) \right\rangle^2 \right\},$$

- Decomposition with respect to local oscillator amplitude, E_{LO} :

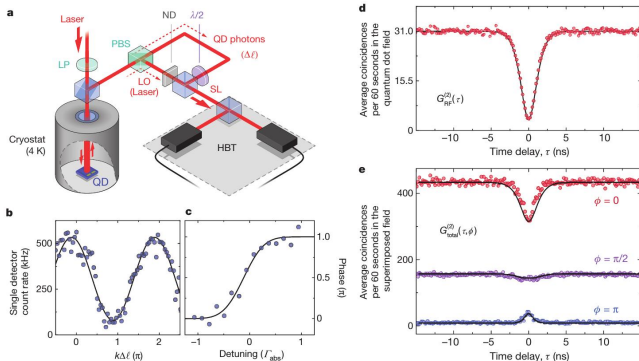
$$\Delta G^{(2,2)} = \sum_{i=0}^4 \Delta G_i^{(2,2)}$$

- Sub-Poisson statistics: $\Delta G_0^{(2,2)} = |T|^4 \left\langle (\Delta \hat{I}_{\text{SI}})^2 \right\rangle$
- Anomalous correlation: $\Delta G_1^{(2,2)} = 2|T|^3 |R| E_{\text{LO}} \left\langle \Delta \hat{E}_{\text{SI}} \Delta \hat{I}_{\text{SI}} \right\rangle$
- Squeezing: $\Delta G_2^{(2,2)} = |T|^2 |R|^2 E_{\text{LO}}^2 \left\langle (\Delta \hat{E}_{\text{SI}})^2 \right\rangle$

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Squeezing in resonance fluorescence

- Proposal of the measurement technique⁹
- Experimental verification¹⁰: Homodyne intensity correlation experiment

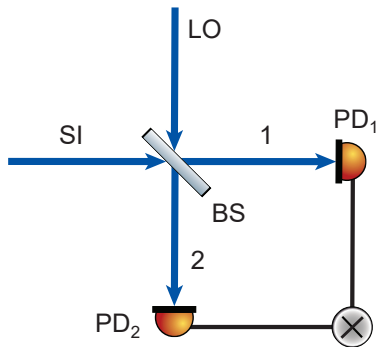


⁹W. Vogel, Phys. Rev. Lett. **67**, 2450 (1991).

¹⁰C. H. H. Schulte, J. Hansom, A. F. Jones, C. Matthiesen, C. Le Gall, and M. Atatüre, Nature **525**, 222 (2015); for experimental details, see supplement.

Homodyne cross-correlation measurement⁶

- The measurement setup:



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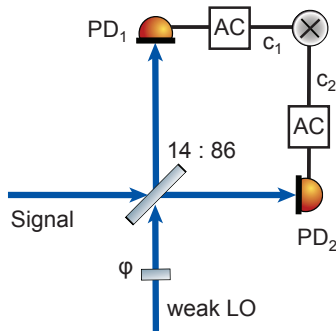
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- Squeezing: $\Delta G_2^{(2,2)} = -|T|^2 |R|^2 E_{\text{LO}}^2 \langle : (\Delta \hat{E}_{\text{SI}})^2 : \rangle$

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Anomalous quantum correlations of squeezed light¹¹

- Homodyning with unbalanced beam splitter and weak LO⁶
- Classical description
- Correlation $C(\phi) = \langle c_1 c_2 \rangle$ of detector current fluctuations
- Separation of different moments:
 $C(\phi) = C_0 + C_1(\phi) + C_2(\phi)$
 - $C_0(\phi) \propto \langle (\Delta I)^2 \rangle$
 - $C_1(\phi) \propto E_L \langle \Delta E_\phi \Delta I \rangle$
 - $C_2(\phi) \propto E_L^2 \langle (\Delta E_\phi)^2 \rangle$
- Experimental result $\det[L(\phi)] < 0$:
 $\langle : \Delta \hat{E}_\phi \Delta \hat{I} : \rangle^2 > \langle : (\Delta \hat{E}_\phi)^2 : \rangle \langle : (\Delta \hat{I})^2 : \rangle$

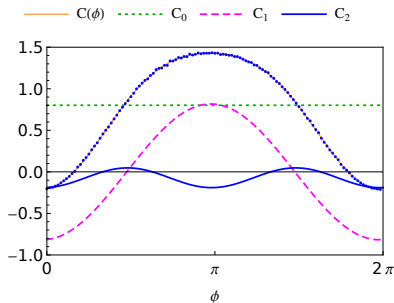


¹¹B. Kühn, W. Vogel, M. Mraz, S. Köhnke, and B. Hage, Phys. Rev. Lett. **118**, 153601 (2017).

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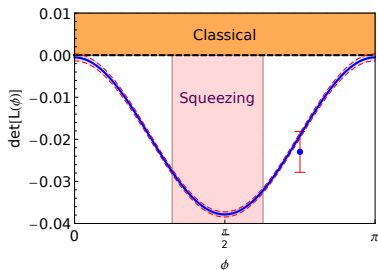
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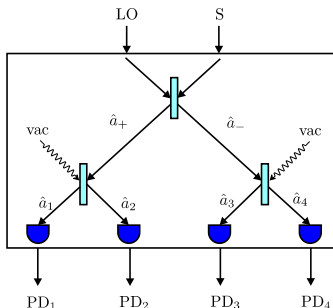
Quantum correlation for large phase interval!

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Balanced homodyne correlation measurement¹²

- Basic measurement device MD_d of depth d with 2^d photodetectors:



Simplest scheme of depth $d = 2$

¹²E. Shchukin and W. Vogel, Phys. Rev. Lett. **96**, 200403 (2006).

Balanced homodyne correlation measurements¹²

- Simplest scenario: one space-time point

$$\mathcal{G}^{(n,m)}(\vec{r}, t) = \langle \hat{\mathcal{E}}^{(-)}(\vec{r}, t)^n \hat{\mathcal{E}}^{(+)}(\vec{r}, t)^m \rangle$$

- Correlation function $\Gamma_l^{(k)}$:

$$\Gamma_l^{(k)} \sim \langle : \hat{N}_+^l \hat{N}_-^{k-l} : \rangle, \quad \hat{N}_\pm = \frac{1}{2}(\hat{\mathcal{E}}^{(-)}\hat{\mathcal{E}}^{(+)} \pm E_{\text{LO}}\hat{\mathcal{X}}_\varphi + E_{\text{LO}}^2)$$

- Balanced data combination:

$$F^{(k)}(\varphi) = \sum_{l=0}^k (-1)^{k-l} \binom{k}{l} \Gamma_l^{(k)} \sim \langle : (\hat{N}_+ - \hat{N}_-)^k : \rangle \sim E_{\text{LO}}^k \langle : \hat{\mathcal{X}}_\varphi^k : \rangle$$

- Fourier analysis (φ dependence):

$$\langle : \hat{\mathcal{X}}_\varphi^k : \rangle = \sum_{l=0}^k \binom{k}{l} \langle \hat{\mathcal{E}}^{(-)l} \hat{\mathcal{E}}^{(+)(k-l)} \rangle e^{i(k-2l)\varphi}$$

⇒ Insensitive to efficiency losses and uncorrelated dark counts!

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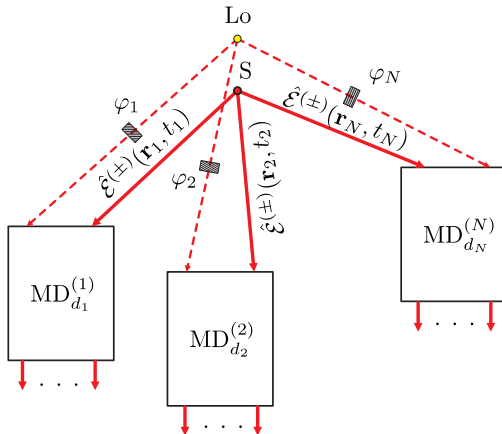
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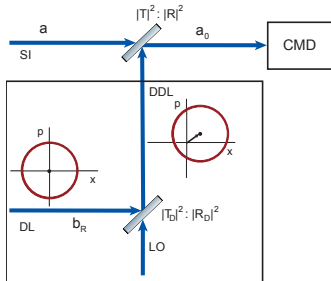
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General correlation measurements

- Extension to many space-time points:



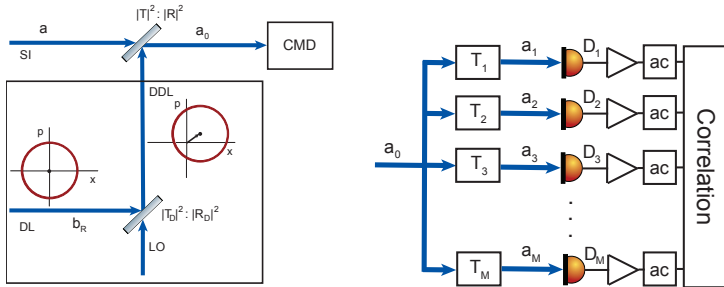
Unbalanced homodyne correlation measurement¹³



- Reference field is a displaced dephased laser (DDL)
- ac correlation of $M = 2m$ detectors yields moments $\langle : [\hat{n}(\alpha)]^m : \rangle$
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- Quantum-state representation via displaced photon-number moments

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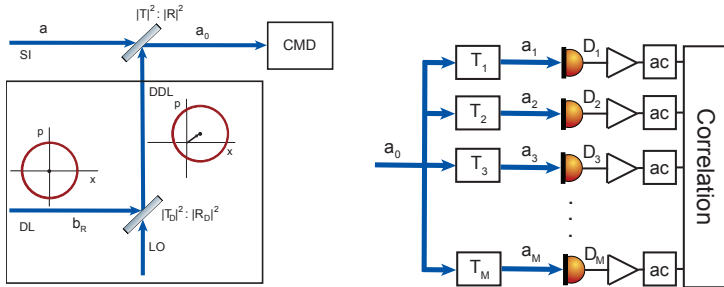
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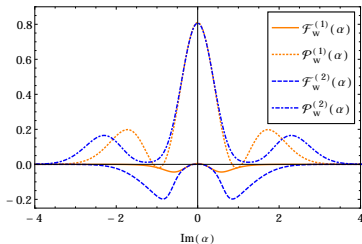
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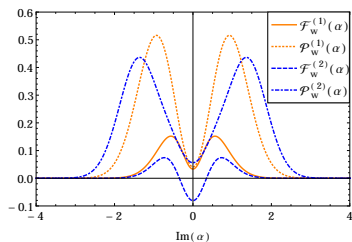
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Vizualizing quantum effects via UHCM¹³



Squeezed vacuum state



Single-photon added thermal state

- Minimal eigenvalue $\mathcal{F}_w^{(k)}(\alpha)$ of matrix of displaced number moments, $L_w(\alpha) = (w^{2(m+m')}\langle: [\hat{n}(\alpha)]^{m+m'} : \rangle)_{mm'}$; for $m, m' = 0, \dots, k$
- Truncated nonclassicality quasiprobabilities, $\mathcal{P}_w^{(k)}(\alpha)$
- $\mathcal{F}_w^{(k)}(\alpha) < 0$ and $\mathcal{P}_w^{(k)}(\alpha) < 0$ certify nonclassicality

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General quantum correlations of light

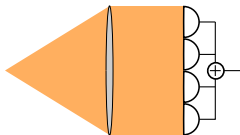
Homodyne correlation measurements

Click counting measurements

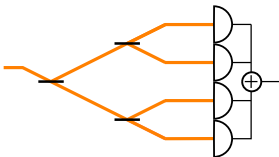
Summary

Click-counting detectors

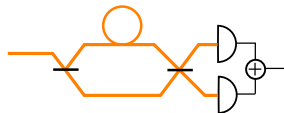
Array detector



Spatial multiplexing



Time-bin multiplexing

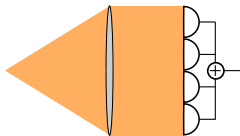


- Split of signal intensity into N smaller ones
- Use of N on-off detectors
- Click-counting probability c_k for $k = 0, \dots, N$ clicks¹⁴

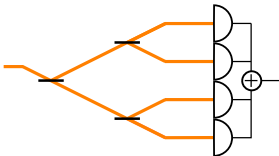
¹⁴Sperling, Vogel, and Agarwal, Phys. Rev. A **85**, 023820 (2012).

Click-counting detectors

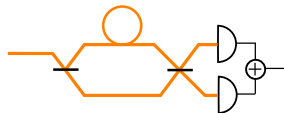
Array detector



Spatial multiplexing



Time-bin multiplexing



- Split of signal intensity into N smaller ones
- Use of N on-off detectors
- Click-counting probability c_k for $k = 0, \dots, N$ clicks¹⁴
- Binomial distribution:

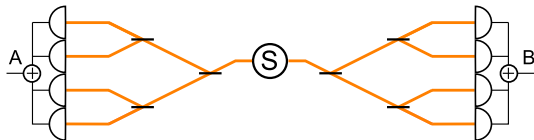
$$c_k = \left\langle : \binom{N}{k} (e^{-\hat{n}/N})^{N-k} (\hat{1} - e^{-\hat{n}/N})^k : \right\rangle$$

¹⁴Sperling, Vogel, and Agarwal, Phys. Rev. A **85**, 023820 (2012).

Experimental implementations (examples)

- Multiplexing schemes:
 - D. Achilles, Ch. Silberhorn, C. Sliwa, K. Banaszek and I. A. Walmsley, Opt. Lett. **28**, 2387 (2003).
 - M. J. Fitch, B. C. Jacobs, T. B. Pittman, and J. D. Franson, Phys. Rev. A **68**, 043814 (2003).
 - G. Zambra, A. Andreoni, M. Bondani, *et al.*, Phys. Rev. Lett. **95**, 063602 (2005).
- Array detectors:
 - E. Waks, E. Diamanti, B. C. Sanders, S. D. Bartlett, and Y. Yamamoto, Phys. Rev. Lett. **92**, 113602 (2004).
 - L. A. Jiang, E. A. Dauler, and J. T. Chang, Phys. Rev. A **75**, 062325 (2007).

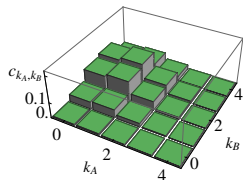
Two-mode click correlations



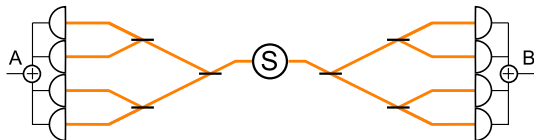
correlation
measurement

$$N_A = N_B = 4$$

- Obtain two-mode click statistics c_{k_A, k_B}



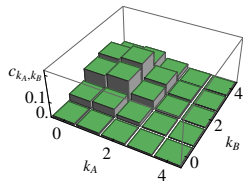
Two-mode click correlations



correlation
measurement

$$N_A = N_B = 4$$

- Obtain two-mode click statistics c_{k_A, k_B}
- Matrix of moments $\mathbf{M} = \langle : \hat{\pi}_A^{m_A + m'_A} \hat{\pi}_B^{m_B + m'_B} : \rangle$
with non-click operator $\hat{\pi}_i = e^{-\hat{n}_i/N}$
- $\mathbf{M} \not\geq 0 \Rightarrow$ quantum correlations¹⁵



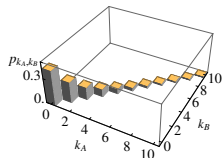
¹⁵Sperling, Vogel, and Agarwal, Phys. Rev. A **88**, 043821 (2013).

Verification of quantum correlations¹⁶

- Two-mode squeezed vacuum state

$$|\xi\rangle = \sum_{n=0}^{\infty} \frac{(\tanh \xi)^n}{\cosh \xi} |n\rangle_A |n\rangle_B$$

| photon-number statistics



¹⁶Sperling, Bohmann, Vogel, Harder, Brecht, Ansari, and Silberhorn, Phys. Rev. Lett. **115**, 023601 (2015).

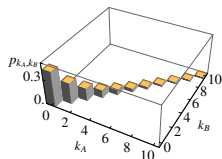
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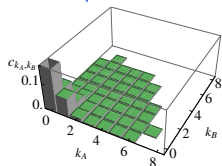
$$|\xi\rangle = \sum_{n=0}^{\infty} \frac{(\tanh \xi)^n}{\cosh \xi} |n\rangle_A |n\rangle_B$$

- Experiment: Silberhorn group (Paderborn)
- Detection: time-bin multiplexing, $N = 8$
- Measured click-counting statistics: c_{k_A, k_B}
- Which properties can be revealed?

| photon-number statistics



| recorded statistics



¹⁶Sperling, Bohmann, Vogel, Harder, Brecht, Ansari, and Silberhorn, Phys. Rev. Lett. **115**, 023601 (2015).

Second-order correlations

Consider moments up to the second order:¹⁶

$$M^{(2,0)} = \begin{pmatrix} 1 & \langle :\hat{\pi}_A: \rangle \\ \langle :\hat{\pi}_A: \rangle & \langle :\hat{\pi}_A^2: \rangle \end{pmatrix}$$

single mode A

$$M^{(0,2)} = \begin{pmatrix} 1 & \langle :\hat{\pi}_B: \rangle \\ \langle :\hat{\pi}_B: \rangle & \langle :\hat{\pi}_B^2: \rangle \end{pmatrix}$$

single mode B

$$M^{(2,2)} = \begin{pmatrix} 1 & \langle :\hat{\pi}_A: \rangle & \langle :\hat{\pi}_B: \rangle \\ \langle :\hat{\pi}_A: \rangle & \langle :\hat{\pi}_A^2: \rangle & \langle :\hat{\pi}_A \hat{\pi}_B: \rangle \\ \langle :\hat{\pi}_B: \rangle & \langle :\hat{\pi}_A \hat{\pi}_B: \rangle & \langle :\hat{\pi}_B^2: \rangle \end{pmatrix}$$

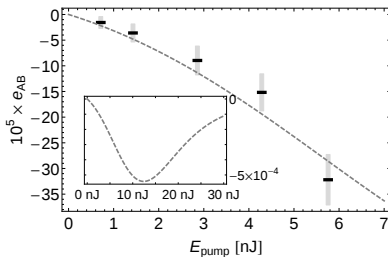
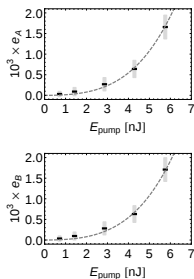
correlations between modes A and B

$M^{(i,j)} \not\geq 0 \Leftrightarrow M^{(i,j)}$ has at least one negative eigenvalue $e_{(i,j)}$

¹⁶Sperling, Bohmann, Vogel, Harder, Brecht, Ansari, and Silberhorn, Phys. Rev. Lett. **115**, 023601 (2015).

Experimental second-order correlations¹⁶

| results



minimal
eigenvalues:

$$e_A > 0 \text{ for } M^{(2,0)}$$

$$e_B > 0 \text{ for } M^{(0,2)}$$

$$e_{AB} < 0 \text{ for } M^{(2,2)}$$

- Single-mode reductions are classical
- No data post-processing needed

⇒ Direct verification of quantum cross correlations

¹⁶Sperling, Bohmann, Vogel, Harder, Brecht, Ansari, and Silberhorn, Phys. Rev. Lett. **115**, 023601 (2015).

Present Section

Introduction

General quantum correlations of light

Homodyne correlation measurements

Click counting measurements

Summary

Summary

- General quantum correlations of light: P functional
- ⇒ General normal- and time-ordered nonclassicality conditions
- Homodyne correlation measurements
 - Homodyne intensity correlations: squeezing in single-atom resonance fluorescence → insensitive to loss and dark counts!
 - Homodyne cross correlation measurement: anomalous correlations of squeezed light → insensitive to loss and dark counts!
 - Balanced homodyne correlation measurement: normal-ordered quadrature moments → not yet implemented!
 - Unbalanced homodyne correlation measurement: displaced number statistics without photon-number resolution → not yet implemented!
 - Click-counting verification of quantum correlations

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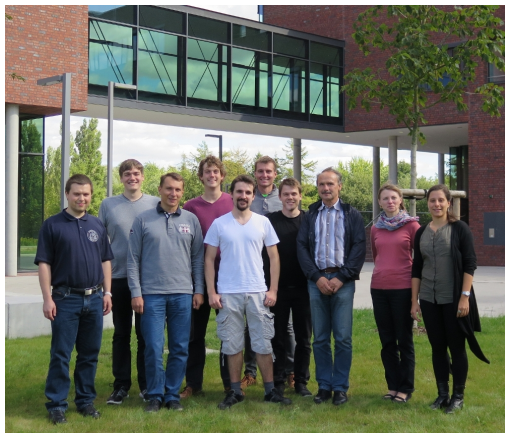
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- Support by EU and DFG:



Thank you for your attention!