

# Theoretical Quantum Optics

## Uncovering Quantum Effects of Light

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Introduction: why quantum light?

Nonclassical states

Determination of quantum states

Uncovering nonclassical phenomena

Uncovering multipartite entanglement

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# Present Section

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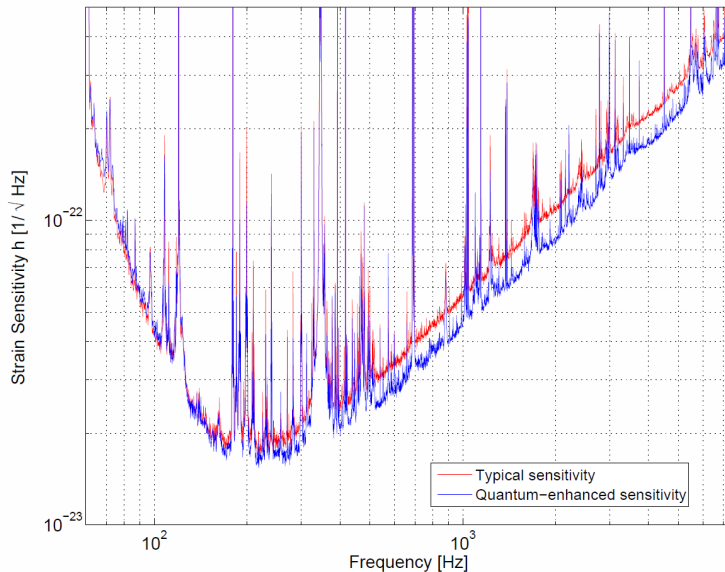
# Ligo gravitational wave interferometer

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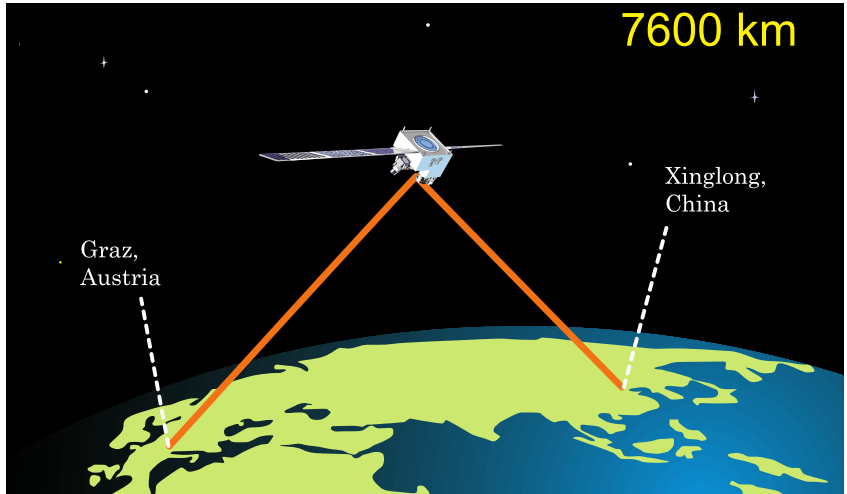


# Quantum metrology with squeezed light



# Secure communication with quantum light

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Determination of quantum states

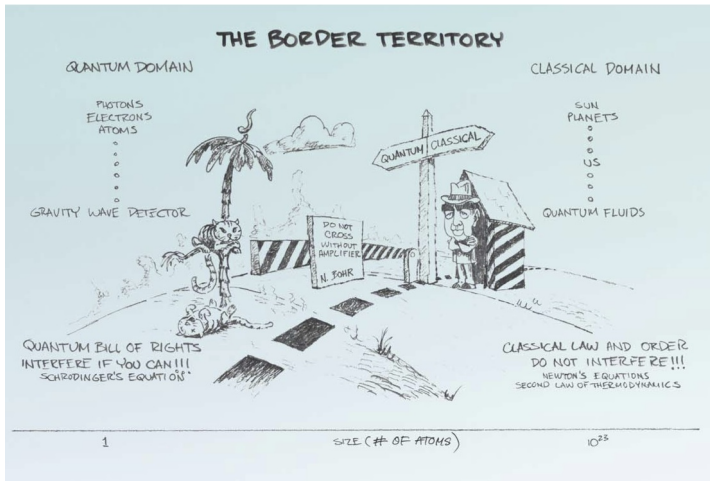
Uncovering nonclassical phenomena

Uncovering multipartite entanglement

Summary

# Nonclassicality: Quantum Superpositions

- Classical reference: coherent states  $|\alpha\rangle$
- Nonclassical state:  $|\psi\rangle = \sum_i c_i |\alpha_i\rangle$



# Classical mixtures versus nonclassical states

- Coherent states  $|\alpha\rangle$ : classical behavior

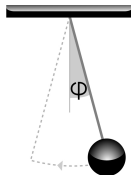
- Mixture of classical states:

$$\hat{\rho}_{\text{cl}} = \sum_i p_i |\alpha_i\rangle\langle\alpha_i| \Rightarrow \int dP_{\text{cl}}(\alpha) |\alpha\rangle\langle\alpha|$$

- General quantum state:<sup>1</sup>  $\hat{\rho} = \int dP(\alpha) |\alpha\rangle\langle\alpha|$

- $P(\alpha) \cong$  quasiprobability:  $P(\alpha) \neq P_{\text{cl}}(\alpha)$

**$P(\alpha)$  is often strongly singular! Experimental determination?**



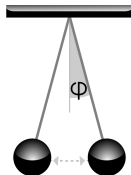
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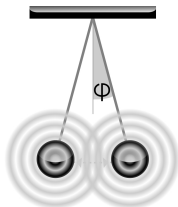


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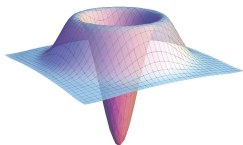
<sup>1</sup>R. J. Glauber, Phys. Rev. **131**, 2766 (1963); E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 227 (1963).

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Experimental  $P$  function<sup>2</sup>

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<sup>2</sup>T. Kiesel, W. Vogel, V. Parigi, A. Zavatta, M. Bellini, Phys. Rev. A **78**, 021804(R) (2008).



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# Experimental determination of $P$ or $W$ functions

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- Characteristic functions of quadratures,

$$\hat{x}_\varphi = \hat{a}e^{i\varphi} + \hat{a}^\dagger e^{-i\varphi}:$$

$$G(k, \varphi) = \langle e^{ik\hat{x}_\varphi} \rangle = \int dx p(x, \varphi) e^{ikx}$$

- Sampling of characteristic function

$$\langle : \hat{D}(\beta) : \rangle = \Phi(ik e^{-i\varphi}) \approx e^{\frac{1}{2}k^2} \frac{1}{N} \sum_{j=1}^N e^{ikx_\varphi(j)}$$

- If possible:  $P(\alpha) = \text{FT}[\Phi(\beta)]$

$$\text{If not: } W(\alpha) = \text{FT}[e^{-\frac{1}{2}|\beta|^2} \Phi(\beta)]$$

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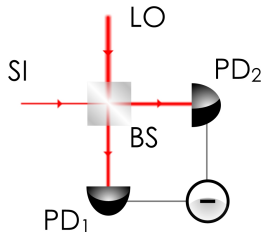
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Balanced homodyne detection

## Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

D. T. Smithey, M. Beck, and M. G. Raymer

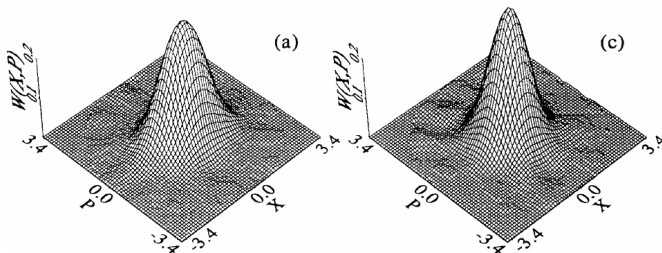
*Department of Physics and Chemical Physics Institute, University of Oregon, Eugene, Oregon 97403*

A. Faridani

*Department of Mathematics, Oregon State University, Corvallis, Oregon 97331*

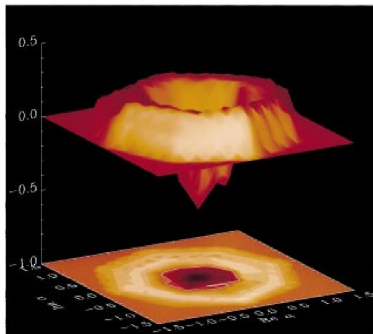
(Received 16 November 1992)

- Wigner function: convolution of  $P(\alpha)$  with Gaussian noise
- Wigner function of a squeezed (left) and vacuum (right) state:  $W \not\equiv 0$



# Reconstruction of motional Fock states

- First demonstration of negative quasiprobability:<sup>3</sup>



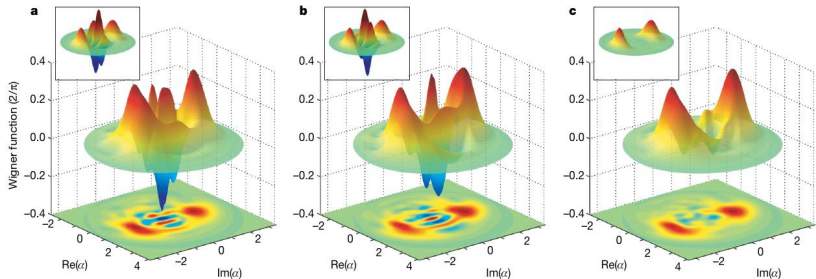
Wigner function of the first motional number state:  $|n = 1\rangle$

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<sup>3</sup>D. Leibfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **77**, 4281 (1996).

# Reconstruction of quantum superpositions

- Reconstruction of quantum superpositions of coherent states:<sup>4</sup>



(a) even coherent state: (b) odd coherent state: (c) Classical mixture:

$$|\alpha\rangle_+ \sim |\alpha\rangle + |-\alpha\rangle$$

$$|\alpha\rangle_- \sim |\alpha\rangle - |-\alpha\rangle$$

$$\hat{\rho}_{cl} \sim |\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$

<sup>4</sup>S. Deleglise, I. Dotsenko, C. Sayrin, J. Bernu, M. Brune, J.-M. Raimond, and S. Haroche, Nature **455** 510 (2008).

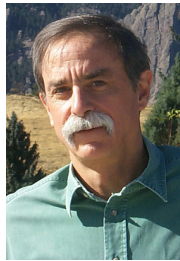
# The Nobel Prize in Physics 2012

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was awarded jointly to



Serge Haroche  
ENS Paris



David J. Wineland  
NIST Boulder, CO

"for ground-breaking experimental methods that enable measuring and manipulation of individual quantum system"

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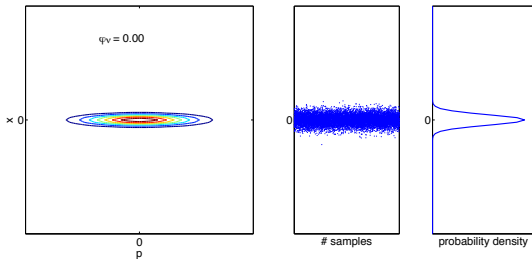
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# Dephased squeezed vacuum state

- Squeezed vacuum state:  $(\mu \hat{a} + \nu \hat{a}^\dagger)|sv\rangle = 0$ ,  $\nu = |\nu|e^{i\varphi_\nu}$

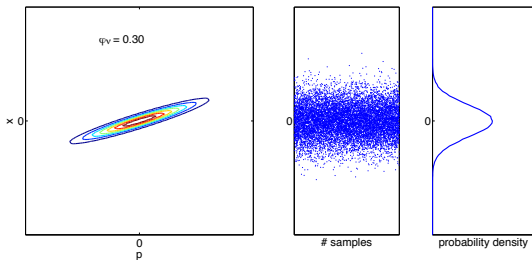


Quantumness: suppression of vacuum noise:  $\langle (\Delta \hat{x}_{\varphi=0})^2 \rangle < \langle (\Delta \hat{x}_{\varphi=0})^2 \rangle_{\text{vac}}$

Do quantum effects survive?

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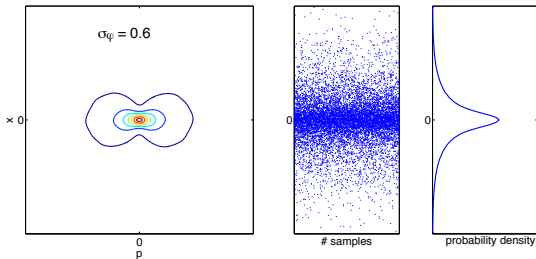


Quantumness: suppression of vacuum noise:  $\langle(\Delta\hat{x}_\varphi)^2\rangle \approx \langle(\Delta\hat{x}_\varphi)^2\rangle_{\text{vac}}$

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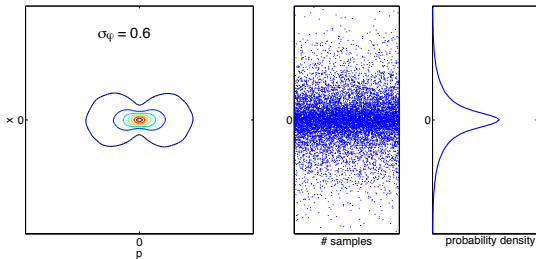


$$\text{Dephasing: } \langle (\Delta \hat{x}_\varphi)^2 \rangle \geq \langle (\Delta \hat{x}_\varphi)^2 \rangle_{\text{vac}}$$

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**Do quantum effects survive?**

# Nonclassicality via characteristic function

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- Characteristic function:

$$\Phi(\beta) = FT[P(\alpha)]$$

- Nonclassicality:<sup>5</sup>

$$|\Phi(\alpha)| > 1$$

- Gaussian phase noise<sup>6</sup>
- Squeezing for  $\sigma < 22.2^\circ$

⇒ Nonclassicality for all  $\sigma$

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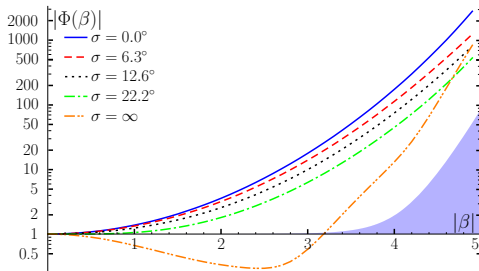
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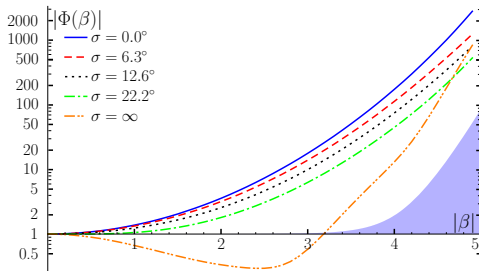
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**Fourier transform of  $\Phi(\beta) \Rightarrow$  strongly singular  $P$  functions!**

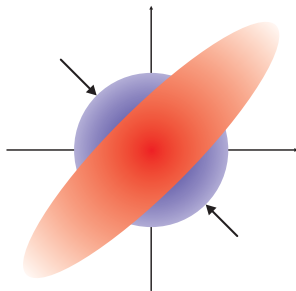
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## $P$ function of squeezed state

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- Squeezing below the vacuum noise level:



- $P$  function of squeezed vacuum  $\Rightarrow$  demanding regularization:

$$P_{\text{sv}}(\alpha) = e^{-\frac{V_X - V_P}{8} \left( \frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \alpha^{*2}} - 2 \frac{V_X + V_P}{V_X - V_P} \frac{\partial}{\partial \alpha} \frac{\partial}{\partial \alpha^*} \right)} \delta(\alpha)$$



# Nonclassicality Quasiprobabilities: $P_\Omega$

- Problem:  $P(\alpha)$  is singular  $\Leftrightarrow \Phi \equiv \text{FT}(P)$  is not integrable
  - Filtering characteristic function:<sup>7</sup>  $\Phi_\Omega(\beta) = \Phi(\beta)\Omega_w(\beta)$
  - Construction of a nonclassicality filter<sup>8</sup>:
    - Rapidly decaying function:  $\omega(\beta) = e^{-|\beta|^4}$
    - Autocorrelation function:  $\Omega_w(\beta) \sim \int \omega(\beta')\omega(\frac{\beta}{w} + \beta')d^2\beta'$
- $\Rightarrow$  Regularized function  $P_\Omega = \text{FT}^{-1}(\Phi_\Omega)$ , called nonclassicality quasiprobability:

**For any quantum state:**  $P_\Omega < 0 \Leftrightarrow P < 0$

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# $P_\Omega$ of Squeezed Vacuum

- Direct sampling of  $P_\Omega$ :<sup>9</sup>

$$P_\Omega(\alpha) \approx \frac{1}{N} \sum_{i=1}^N f_\Omega(x_i, \varphi_i; \alpha, w)$$

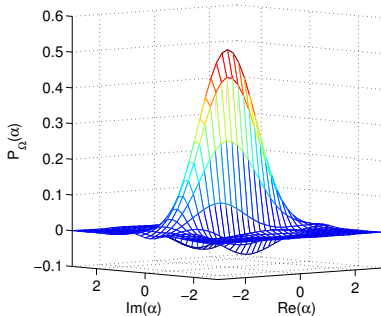
- Pattern function:

$$f_\Omega(x, \varphi; \alpha, w) = F[\Omega_w(b)]$$

- Phase locked measurement with interpolations

- Continuous phase measurement<sup>10</sup>

⇒ Result for  $P_\Omega$



Squeezed vacuum state<sup>9</sup>

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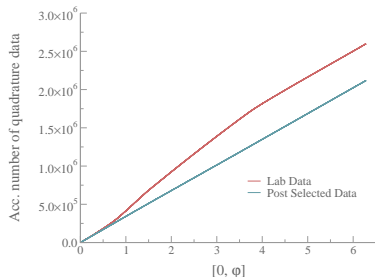
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Quantum random numbers

<sup>9</sup>T. Kiesel, W. Vogel, B. Hage, R. Schnabel, Phys. Rev. Lett. **107**, 113604 (2011).

<sup>10</sup>E. Agudelo, J. Sperling, W. Vogel, S. Köhnke, M. Mraz, and B. Hage, Phys. Rev. A **92**, 033837 (2015).

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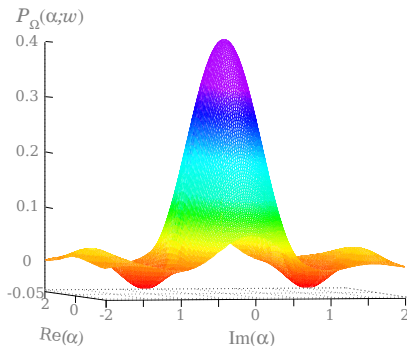
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# Quantum Entanglement

## Example: Schrödinger's cat (1935)

- Classical reference: product state  $|\Psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$
- Cat state:



# Quantum Entanglement

## Example: Schrödinger's cat (1935)

- Classical reference: product state  $|\Psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$
- Cat state:  $|\Psi\rangle \sim |\text{atom}\rangle \otimes |\text{cat alive}\rangle + |\text{atom decayed}\rangle \otimes |\text{cat dead}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$



# Classical Correlation versus Quantum Entanglement

- Uncorrelated (product) states:  $|a, b\rangle \equiv |a\rangle \otimes |b\rangle$
- Mixture of uncorrelated states  $\Rightarrow$  separable states:<sup>11</sup>

$$\hat{\sigma} = \sum_i p_i |a_i, b_i\rangle \langle a_i, b_i| \quad (p_i: \text{probability})$$

$$\Rightarrow \int dP_{\text{cl}}(a, b) |a, b\rangle \langle a, b| \quad (P_{\text{cl}}: \text{joint probability})$$

- General state:  $\hat{\rho} = \int dP(a, b) |a, b\rangle \langle a, b|$
- Entanglement quasiprobability:<sup>12</sup>

$$P(a, b) \neq P_{\text{cl}}(a, b)$$



<sup>11</sup>R. F. Werner, Phys. Rev. A **40**, 4277 (1989).

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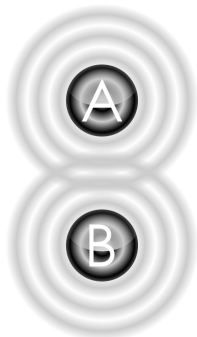
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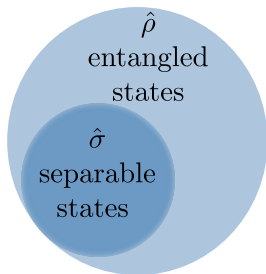
# Entanglement Witnesses

- Separable states form a convex set
- Exists hyperplane,  $\langle \hat{W} \rangle = 0$ , dividing set in two parts;  $\hat{W}$ : Witness operator<sup>13</sup>
- Systematic construction of optimal multipartite entanglement witnesses:<sup>14</sup>

- Hermitian operator  $\hat{L}$
- Separability eigenvalue problem for  $N$  partitions:

$$\hat{L}_{a_1, \dots, a_{j-1}, a_{j+1}, \dots, a_N} |a_j\rangle = g |a_j\rangle, \\ \text{for } j = 1, \dots, N$$

$$\Rightarrow \hat{W}_{\text{opt}} = \hat{L} - \inf(g) \hat{1}$$



<sup>13</sup>M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A **232**, 1 (1996).

<sup>14</sup>J. Sperling and W. Vogel, Phys. Rev. Lett. **111**, 110503 (2013).

# Entanglement Witnesses

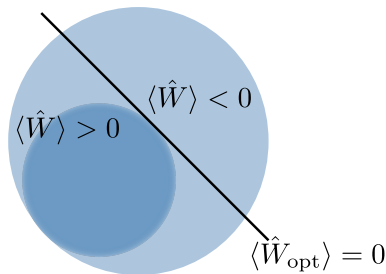
- Separable states form a convex set
- Exists hyperplane,  $\langle \hat{W} \rangle = 0$ , dividing set in two parts;  $\hat{W}$ : Witness operator<sup>13</sup>
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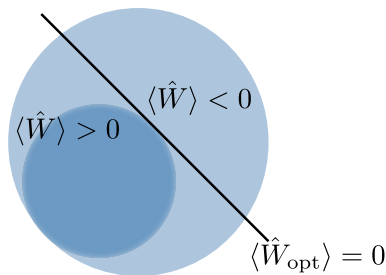
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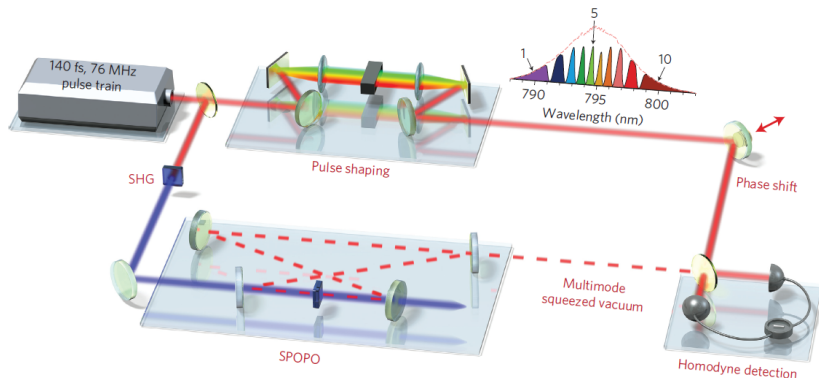


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<sup>14</sup>J. Sperling and W. Vogel, Phys. Rev. Lett. **111**, 110503 (2013).

# Continuous variable Gaussian entanglement

Synchronously pumped optical parametric oscillator (SPOPO):<sup>15</sup> frequency comb laser. Spectrum divided into elements of equal energy.



<sup>15</sup>J. Roslund, R. Medeiros de Araújo, S. Jiang, C. Fabre, and N. Treps, Nature Photon. **8**, 109 (2014).



## Wavelength-multiplexed quantum networks with ultrafast frequency combs

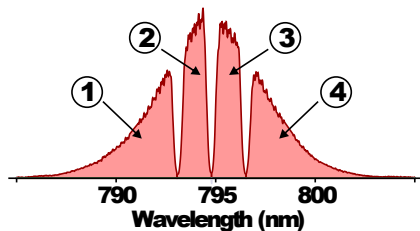
Jonathan Roslund, Renné Medeiros de Araújo, Shifeng Jiang, Claude Fabre and Nicolas Treps\*

Highly entangled quantum networks (cluster states) lie at the heart of recent approaches to quantum computing<sup>1,2</sup>. Yet the current approach for constructing optical quantum networks does so one node at a time<sup>3–5</sup>, which lacks scalability. Here, we demonstrate the single-step fabrication of a multimode quantum resource from the parametric downconversion of femtosecond-frequency combs. Ultrafast pulse shaping<sup>6</sup> is employed to characterize the comb's spectral entanglement<sup>7,8</sup>. Each of the 511 possible bipartitions among ten spectral regions is shown to be entangled; furthermore, an eigenmode

The coupling strength between modes at frequencies  $\omega_m$  and  $\omega_n$  is dictated by the matrix  $L_{m,n} = f_{m,n} \cdot p_{m+n}$ , where  $f_{m,n}$  is the phase-matching function<sup>14,15</sup> and  $p_{m+n}$  is the pump spectral amplitude at frequency  $\omega_m + \omega_n$  (ref. 16).

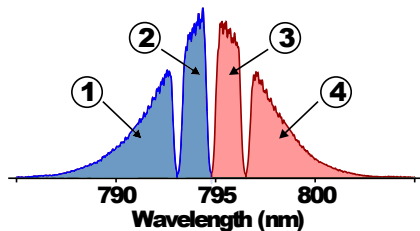
**Frequency entanglement.** A femtosecond pulse train is produced with a mode-locked titanium-sapphire oscillator delivering  $\sim 140$  fs pulses, and its second harmonic serves to pump synchronously a below-threshold OPO, as detailed in Fig. 1. Homodyne detection coupled with ultrafast pulse shaping is then employed

# Entanglement beyond bipartitions



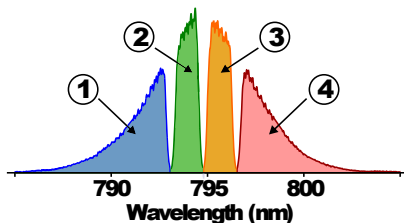
$$\left( \begin{array}{cccc} \{1, 2, 3, 4\} & \{1, 2, 3\}:\{4\} & & \\ \{1, 2, 4\}:\{3\} & \{1, 2\}:\{3, 4\} & \{1, 2\}:\{3\}:\{4\} & \\ \{1, 3, 4\}:\{2\} & \{1, 3\}:\{2, 4\} & \{1, 3\}:\{2\}:\{4\} & \\ \{1, 4\}:\{2, 3\} & \{1\}:\{2, 3, 4\} & \{1\}:\{2, 3\}:\{4\} & \\ \{1, 4\}:\{2\}:\{3\} & \{1\}:\{2, 4\}:\{3\} & \{1\}:\{2\}:\{3, 4\} & \{1\}:\{2\}:\{3\}:\{4\} \end{array} \right)$$

# Entanglement beyond bipartitions



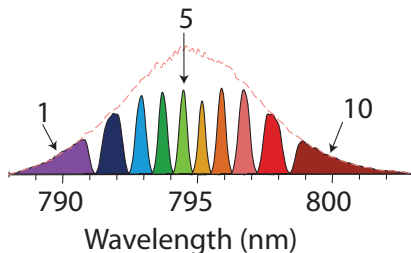
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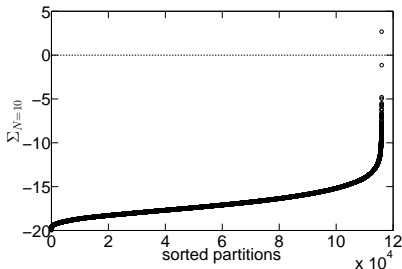
# Entanglement beyond bipartitions



- Entanglement for any mode partition
- 4 mode states: 15 partitions
- 10 mode states: 511 bipartitions, but **115975** partitions  $\Rightarrow$  rich structure

# Full entanglement test of a 10-mode state<sup>16</sup>

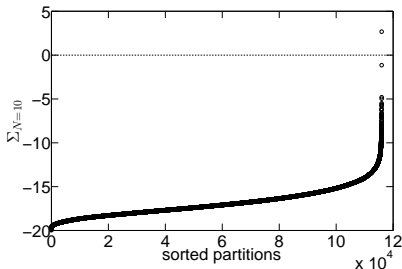
- Entanglement test: 10-mode frequency comb state
- Absolute value of  $\Sigma$  is lower bound for significance of entanglement test
- Negative  $\Sigma$  value: state is entangled with respect to chosen partition
- Full entanglement of complex system: all 115974 nontrivial partitions!



<sup>16</sup>S. Gerke, J. Sperling, W. Vogel, Y. Cai, J. Roslund, N. Treps, and C. Fabre, Phys. Rev. Lett. **114**, 050501 (2015).

## Full entanglement test of a 10-mode state<sup>16</sup>

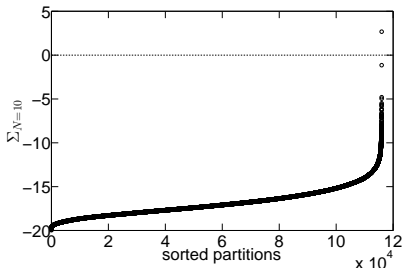
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# Present Section

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Introduction: why quantum light?

Nonclassical states

Determination of quantum states

Uncovering nonclassical phenomena

Uncovering multipartite entanglement

**Summary**

- **Notions of nonclassicality and entanglement**
  - Reconstructions of quantum states
  - Uncovering nonclassical states
  - Nonclassicality quasiprobabilities
- ⇒ Direct sampling; application to squeezed light
- Uncovering multipartite entanglement: all 115974 partitions
  - Support by EU and DFG:

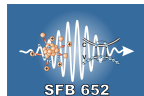
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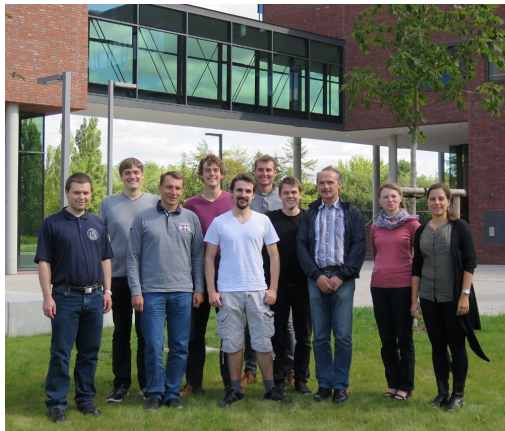
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**Thank you for your attention!**