Summer School on Modern Quantum Technologies, BITP, Kiev
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Superconducting qubits

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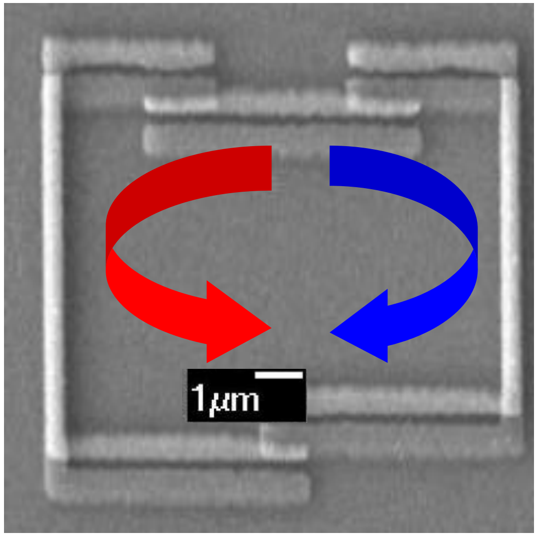
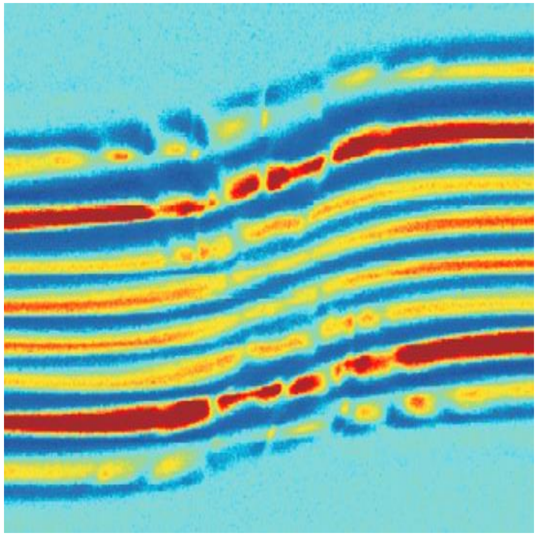




AGENDA

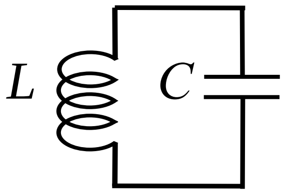
Now we have to have:
(1) Lecture
(2) Tutorials

We will not consider these separately.
We will now rather have the *superposition* of (1) and (2),
in the spirit of Newton's "*When studying science, the examples are more useful than the rules*".

- **Current-biased junction => phase qubit**
- **Superconducting island => charge qubit**
- **Ring with junctions => flux qubit**

- **and dynamic phenomena in (superconducting) qubits: Landau-Zener-Stückelberg-Majorana and multi-photon transitions**

SUPERCONDUCTING QUBITS: WE NEED NON-LINEAR INDUCTANCE



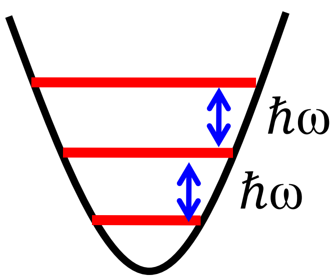
Linear inductance
$L = \Phi / I$

$$H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2$$

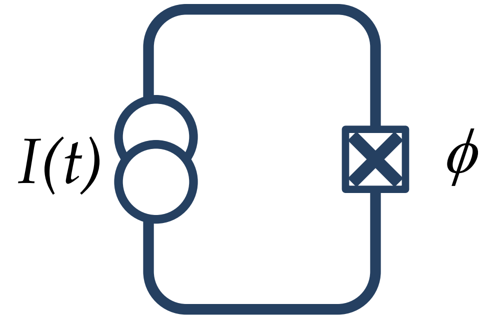
$$(V = \dot{\Phi},\, I = \Phi / L)$$

$$x = V,\, m = C,\, \omega^2 = 1 / LC$$

$$H = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega^2 x^2$$



The only dissipationless non-linear element is the Josephson contact



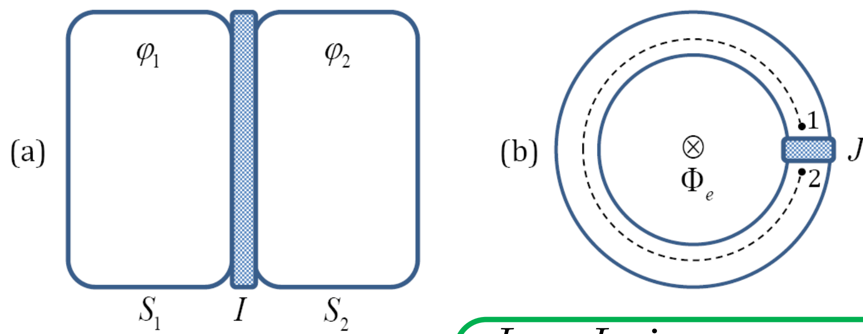
$$I_J = I_c \sin\phi$$

It has non-linear inductance

$$L_J = \frac{d\Phi}{dI} = \frac{\Phi_0}{2\pi I_c \cos\phi}$$

$$\phi = 2\pi\Phi / \Phi_0$$

JOSEPHSON EFFECT – summary



(a) The current through the junction is parameterized by the phase difference:

$$\boxed{I_J = I_c \sin\varphi, \qquad \varphi = \varphi_1 - \varphi_2; \\ V = \frac{\hbar}{2e}\dot{\varphi}.}$$

(b) Consider how this can be controlled by the magnetic flux.
First, the current density of the Cooper pairs:

$$\mathbf{j}_s = (2e)\left(\Psi^* \frac{\mathbf{p} - 2e\mathbf{A}/c}{2(2m)}\Psi + c.c.\right) = -i\frac{e\hbar}{2m}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - \frac{2e^2}{mc}|\Psi|^2\mathbf{A} =$$

$$= 2e|\Psi|^2 \frac{\hbar\nabla\varphi - 2e\mathbf{A}/c}{2m} \equiv 2en_s\mathbf{v}_s.$$

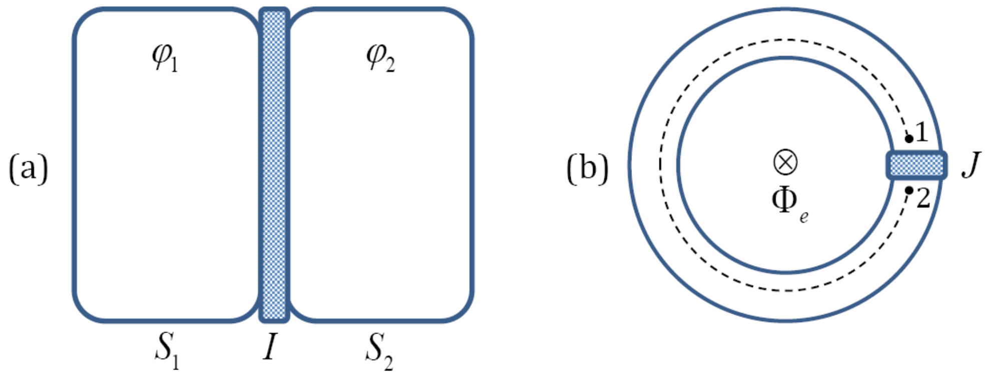
$$\Psi(\mathbf{r}) = |\Psi(\mathbf{r})|e^{i\varphi(\mathbf{r})} = \sqrt{n_s}e^{i\varphi(\mathbf{r})}$$

We rewrite, $\mathbf{j}_s = 2en_s\dfrac{\hbar\nabla\varphi - 2e\mathbf{A}/c}{2m}$, consider the ring in (b), and integrate ($j_s = 0$):

$$\hbar\int_1^2 d\mathbf{l}\nabla\varphi = \hbar\left(\varphi_2 - \varphi_1\right) \equiv -\hbar\varphi \triangleq \frac{2e}{c}\int_1^2 d\mathbf{l}\mathbf{A} \approx \frac{2e}{c}\oint d\mathbf{l}\mathbf{A} = \frac{2e}{c}\Phi_e.$$

So, we obtained: $\boxed{\varphi = 2\pi\dfrac{\Phi_e}{\Phi_0}, \qquad \Phi_0 = \dfrac{hc}{2|e|}.}$

JOSEPHSON EFFECT – summary



* The current through the junction is parameterized by the phase difference:

$$I_J = I_c \sin\varphi, \quad \varphi = \varphi_1 - \varphi_2;$$

$$V = \frac{\hbar}{2e}\dot{\varphi}.$$

* The phase difference can be controlled by the external magnetic flux

$$\varphi = 2\pi\frac{\Phi_e}{\Phi_0}, \quad \Phi_0 = \frac{hc}{2|e|}.$$

* From the definition $V = L(dI/dt)$ we have the Josephson inductance

$$\frac{dI_J}{dt} = I_c \cos\phi \frac{d\phi}{dt} = \frac{2eI_c}{\hbar}\cos\phi \cdot V \equiv \frac{V}{L_J}.$$

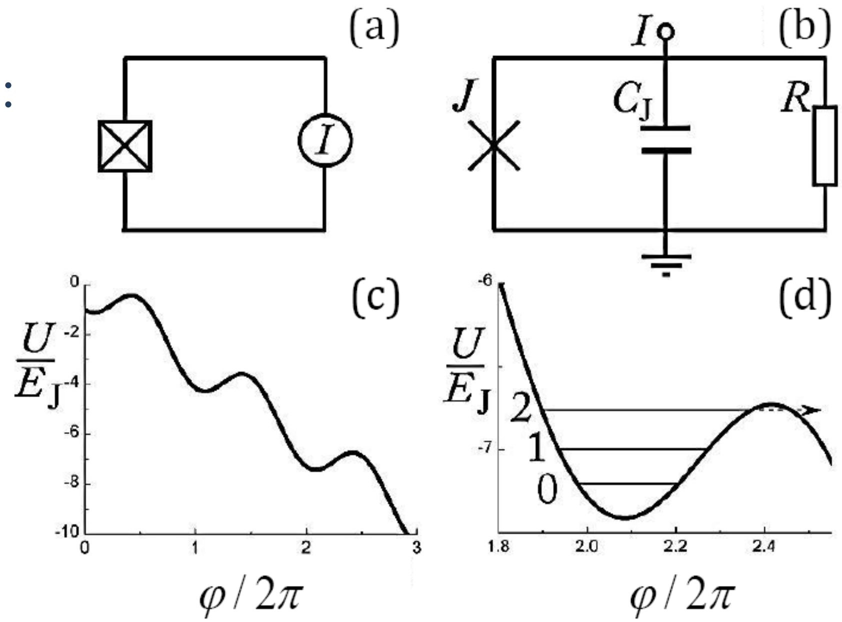
* Josephson energy: $E(\varphi) = \int I_J V dt = E_J\left(1-\cos\varphi\right), \quad E_J = \dfrac{\hbar I_c}{2|e|}.$

* And the energy, associated with the charge Q on the C_J capacitor plate $(Q = C_J V)$:

$C_J V^2/2 = Q^2/2C_J$. Per one electron this gives characteristic charging energy: $E_C = e^2/2C_J$.

CURRENT-BIASED JUNCTION: description

The junction (a) and its equivalent circuit (b):



The Kirchhoff law for the junction:

$$C_J \frac{dV}{dt} + \frac{V}{R} + I_J = I.$$

Or, using the Josephson relations:

$$\frac{\hbar C_J}{2e}\frac{d^2\varphi}{dt^2} + \frac{\hbar}{2eR}\frac{d\varphi}{dt} + I_c \sin\varphi = I.$$

Let's multiply this with $\hbar / 2e$
and introduce obvious notations =>

$$m\ddot{\varphi} + \lambda\dot{\varphi} = -\frac{dU}{d\varphi},$$
$$U(\varphi) = -E_J(\cos\varphi + \varphi I / I_c).$$

This corresponds to the mechanical motion with the washboard potential, with local minima at $I < I_c$.

CURRENT-BIASED JUNCTION: Lagrangian

Continuing the mechanical analogy, we can write down the Lagrangian and the Hamiltonian and quantize the system. For simplicity we neglect here the dissipation, the smallness of which is necessary for practical applications.

The electrostatic energy plays the role of the kinetic energy:

$$K = m\dot{\varphi}^2 / 2 = (\hbar^2 / 16E_C)\dot{\varphi}^2.$$

The Josephson energy plays the role of the potential energy:

$$U(\varphi) = -E_J(\cos\varphi + \varphi I / I_c).$$

Then, indeed, with the Lagrangian $L = K - U$,

the Lagrange equation $\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} = \frac{\partial L}{\partial \varphi}$

gives the motion equation from the previous slide.

Canonical momentum, conjugated to the canonical coordinate φ, is

$$p = \frac{\partial L}{\partial \dot{\varphi}} = m\dot{\varphi} = \frac{\hbar}{2e} C_J V = \hbar \frac{Q}{2e} = \hbar n.$$

CURRENT-BIASED JUNCTION: quantization

Then, we obtain the Hamiltonian:

$$H(p,\varphi) = p\dot{\varphi} - L = \frac{p^2}{2m} + U = 4E_C n^2 - E_J\left(\cos\varphi + \varphi\frac{I}{I_c}\right).$$

Quantization: $\varphi \to \hat{\varphi}, \quad p \to \hat{p} = -i\hbar\frac{\partial}{\partial\varphi} \quad \Leftrightarrow n \to \hat{n} = -i\frac{\partial}{\partial\varphi}.$

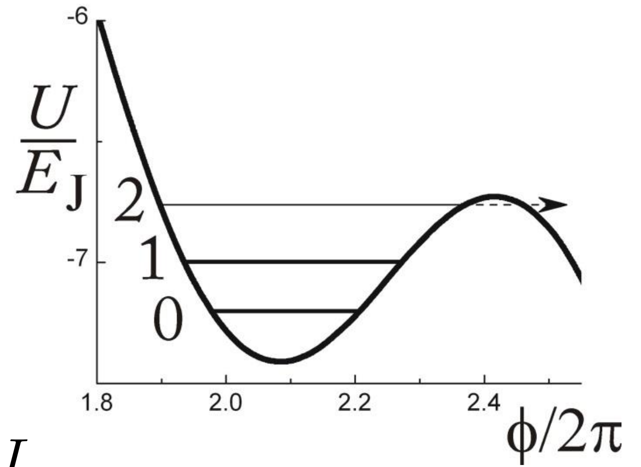
The commutation relation: $[\varphi, p] = i\hbar \Rightarrow [\varphi, n] = i,$

which results in the relation for fluctuations: $\Delta n \Delta\varphi \geq 1.$

Respectively, for $E_C \gg E_J$, a well defined value is the charge, $\Delta n \ll n$.

Then, in this, and in the opposite case, we would have the **charge** and **phase** (or **flux**, in the geometry of an interferometer) qubits.

CURRENT-BIASED JUNCTION => phase qubit



The quantization results in the appearance of discrete energy levels in the potential with local minima. Importantly, with non-equidistant levels.

For the description of the phase qubit, we approximate the potential U by a parabola, expanding it near the minimum, where

$$U' = 0 = E_J(\sin\varphi - I/I_c) \quad \Rightarrow \varphi = \varphi_0 = \arcsin I/I_c.$$

Then, omitting the constant term, we have

$$U \approx E_J \cos\varphi_0 \frac{(\varphi - \varphi_0)^2}{2} \equiv m\omega_q^2 \frac{(\varphi - \varphi_0)^2}{2},$$

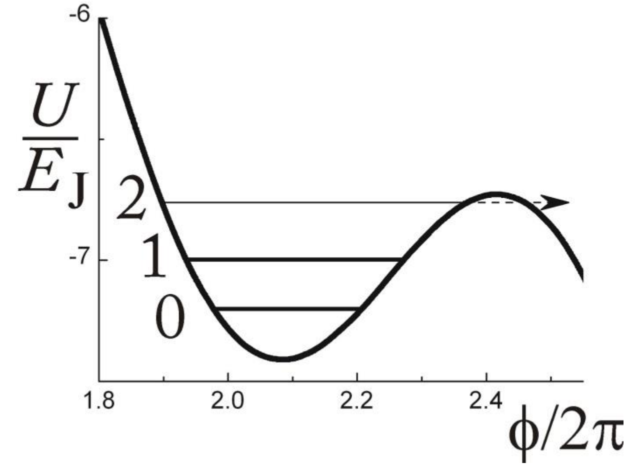
$$\omega_q^2 = \frac{E_J \cos\varphi_0}{m} = \frac{8E_J E_C}{\hbar^2}\sqrt{1 - \left(\frac{I}{I_c}\right)^2}.$$

The energy levels in such harmonic potential: $E_k = \hbar\omega_q(k + 1/2)$.

So, the distance between the qubit energy levels: $\Delta E = E_1 - E_0 = \hbar\omega_q$.
And this is defined by the bias current, $\omega_q = \omega_q(I)$.

PHASE QUBIT: operation

The current is chosen so that to have 3 energy levels, for operation and for the read-out.



The qubit can be controlled by applying pulses resonant with the qubit frequency, defined by

$$\Delta E = E_1 - E_0 = \hbar\omega_q.$$

The probability of tunneling from this levels = 0, hence

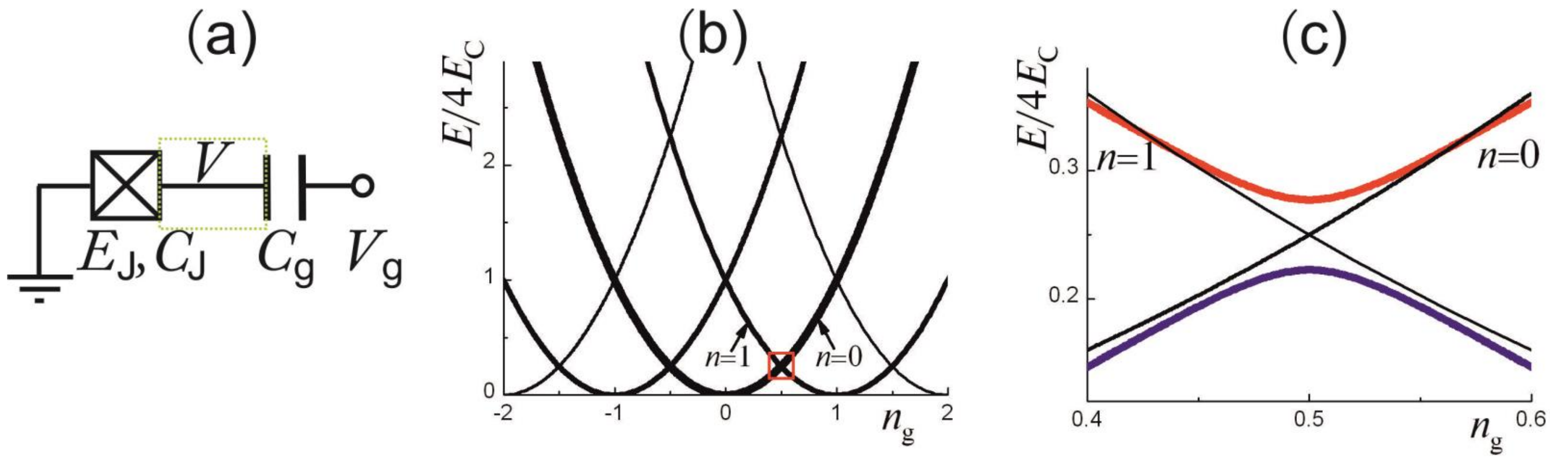
$$\bar{\varphi} = const, \quad V = \frac{\hbar}{2e}\dot{\bar{\varphi}} = 0.$$

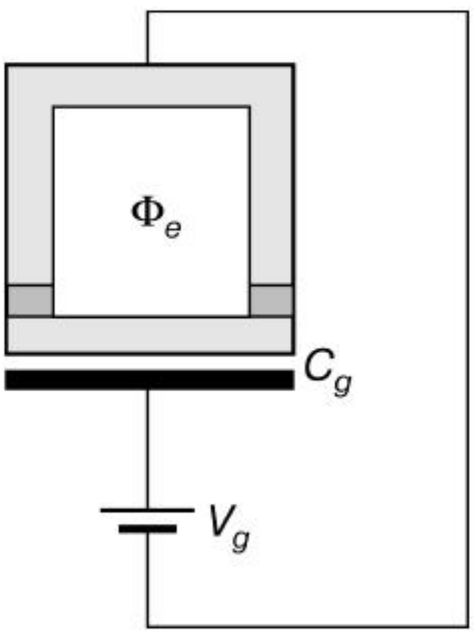
Then the measurement is done by applying the frequency equal to

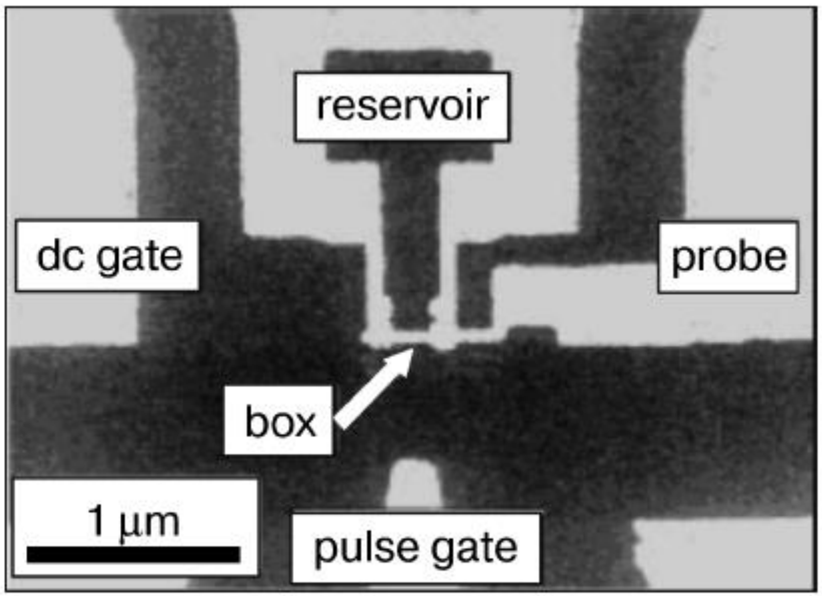
$$\omega_{21} = \left(E_2 - E_1\right) / \hbar > \omega_q.$$

If the qubit was in the excited state, then we observe the voltage pulse $V = \dfrac{\hbar}{2e}\dot{\varphi}.$

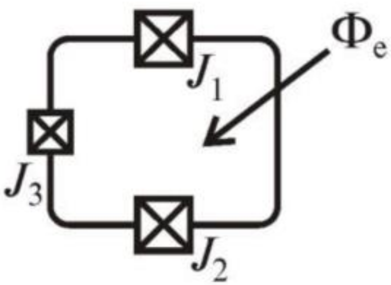
SUPERCONDUCTING ISLAND => CHARGE QUBIT

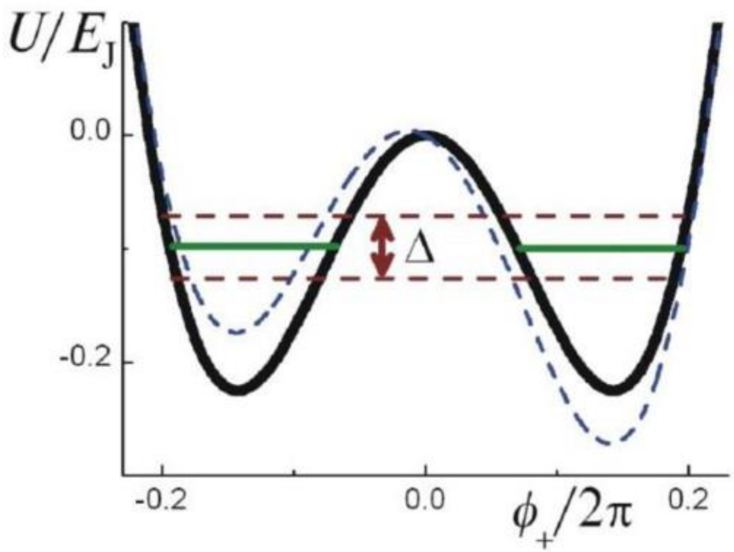


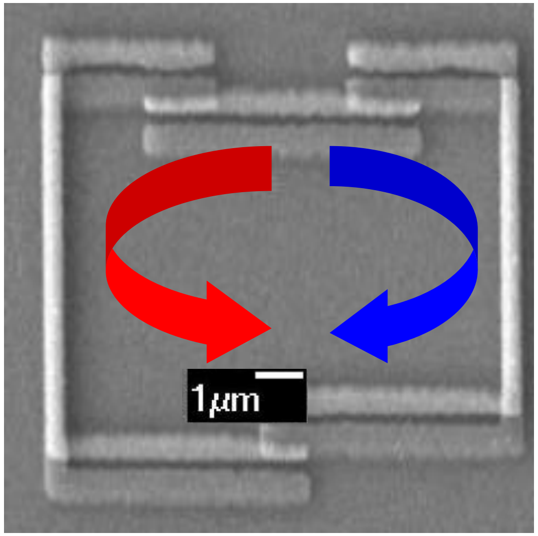




RING WITH JUNCTIONS => FLUX QUBIT







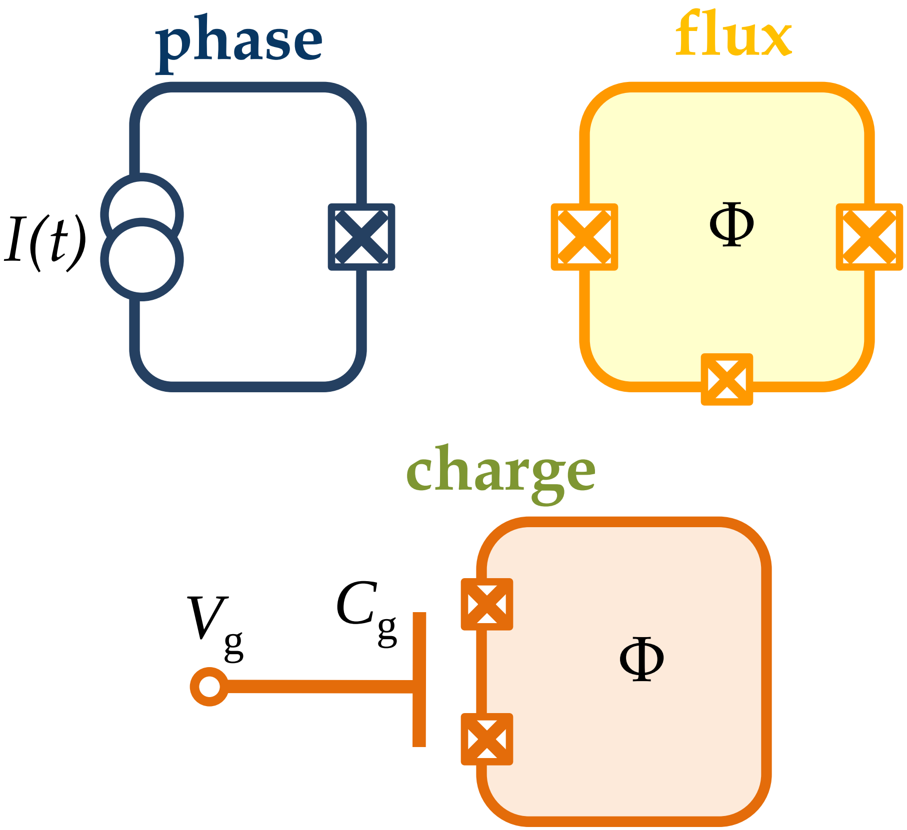
How to excite qubits?

We need the resonant pulse: $E = h\nu = \hbar\omega$.

reCAPTCHA?

No. It is one of the nice
artistic objects here.
See there

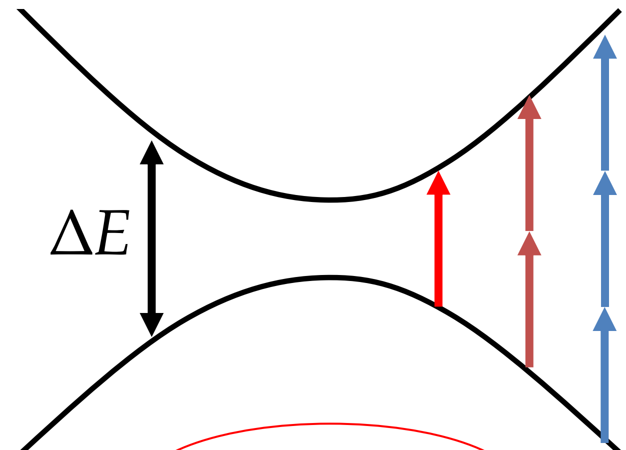
Josephson / superconducting qubits



variable parameters

$$\varepsilon(t) = \varepsilon_0 + A \sin \omega t$$

$$H = -\frac{\Delta}{2}\sigma_x - \frac{\varepsilon(t)}{2}\sigma_z \qquad \Delta E = \sqrt{\Delta^2 + \varepsilon_0^2}$$

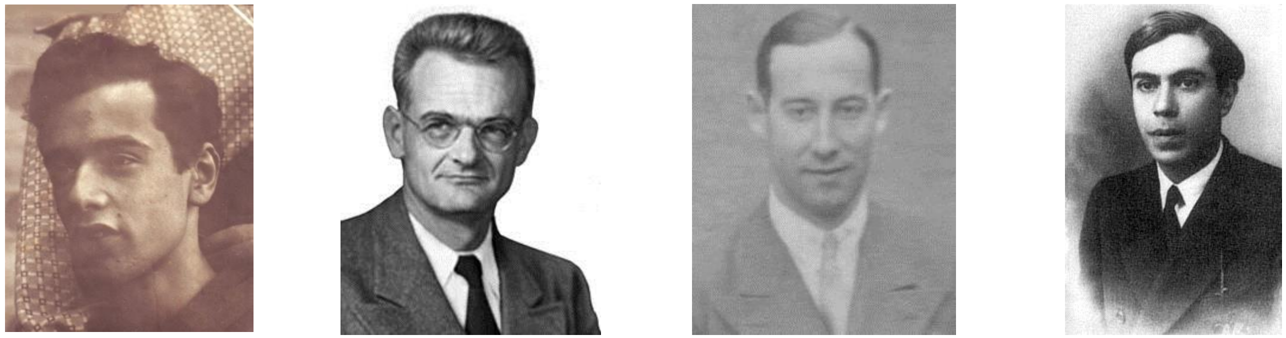


$$\Delta E \approx k \cdot \hbar\, \omega$$

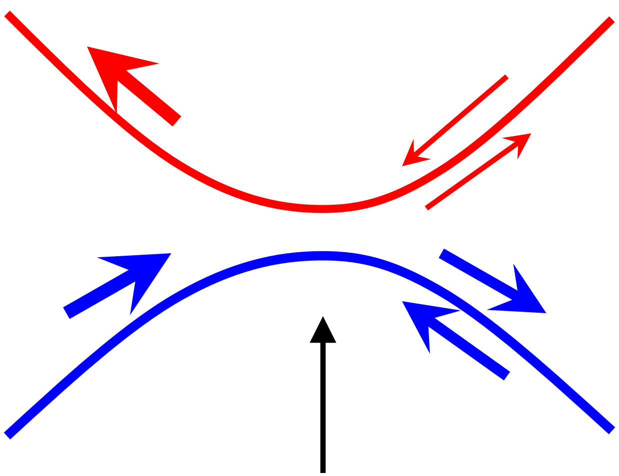
spectroscopy

control

Landau-Zener-Stückelberg-Majorana transition



In 1932, these four scientists, then under thirty, from four different countries, did very closely related works on the transitions in two-level systems.

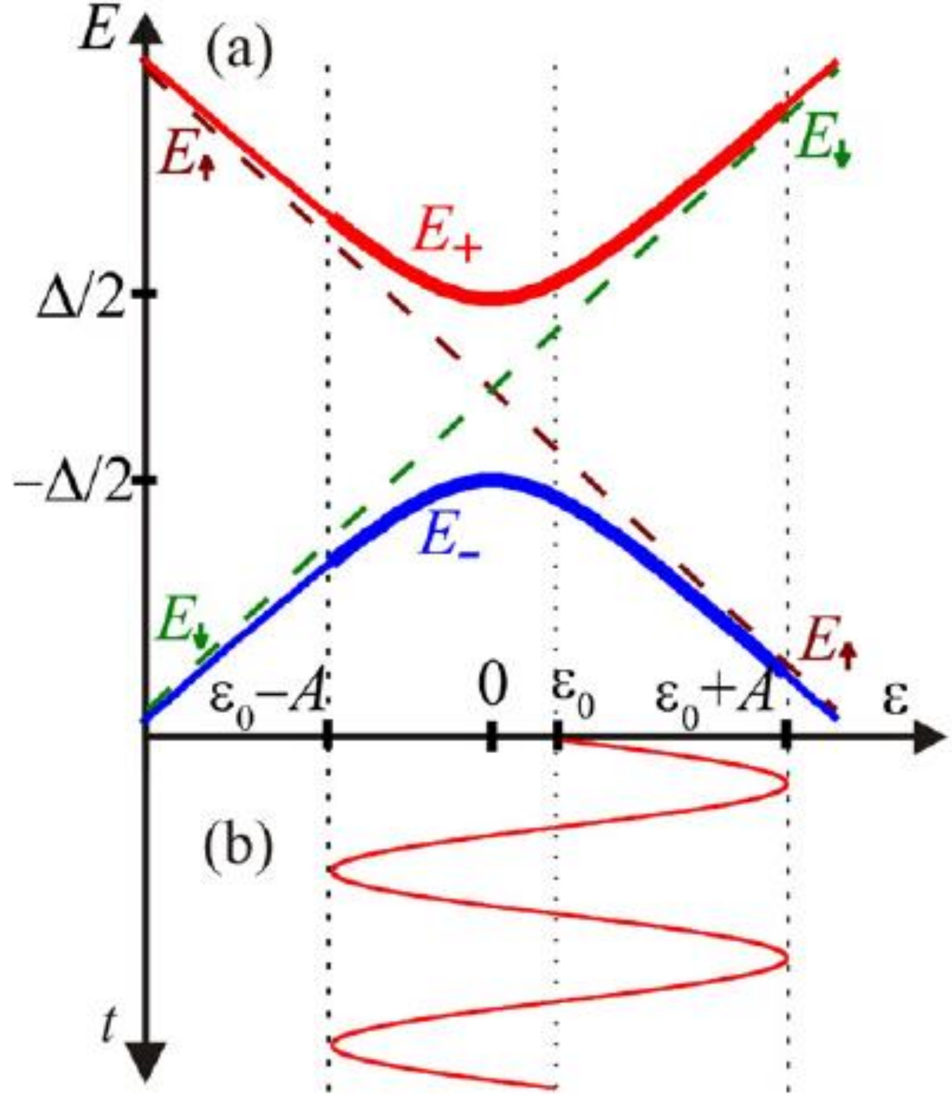


Avoided-level crossing, which is passed twice during the period $T = 2\pi / \omega$.

A driven two-level system experiences a transition to a nearby level with certain probability (given by the LZ formula); while for the repetitive process, a **(Stückelberg) phase** is accumulated, which results in **quantum interference**.

Formulation of the problem

[Shevchenko, Ashhab, Nori, Phys. Rep. (2010)]



Hamiltonian of a two-level system:

$$H = -\frac{\Delta}{2}\sigma_x - \frac{\varepsilon(t)}{2}\sigma_z$$

with time-dependent bias:

$$\varepsilon(t) = \varepsilon_0 + A\sin\omega t.$$

Diabatic eigenenergies:

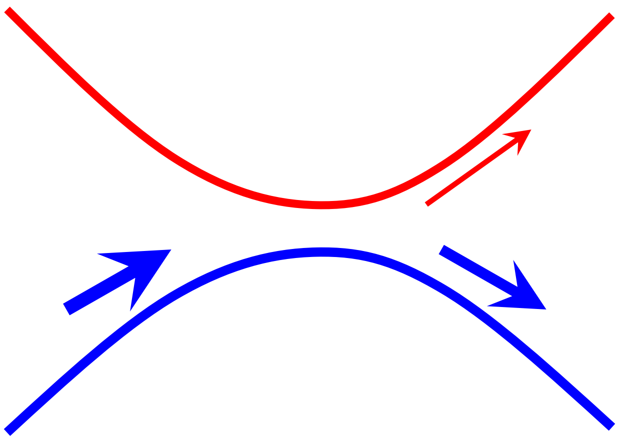
$$\boxed{E_{\downarrow\uparrow} = \pm\frac{1}{2}\varepsilon_0}$$

A-diabatic eigenenergies:

$$\boxed{E_\pm = \pm\frac{1}{2}\sqrt{\varepsilon(t)^2 + \Delta^2}}$$

<u>Task is to find the upper-level occupation probability.</u>

Single passage: Landau-Zener transition, like a beam-splitter



$$P_+ = P_{LZ}$$

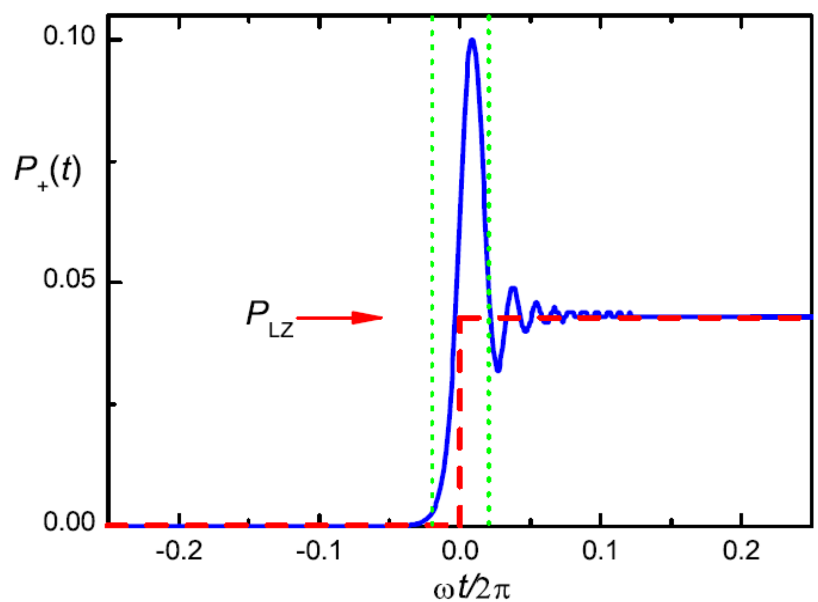
$$P_-(t=0) = 1$$

Landau-Zener (LZ) formula:

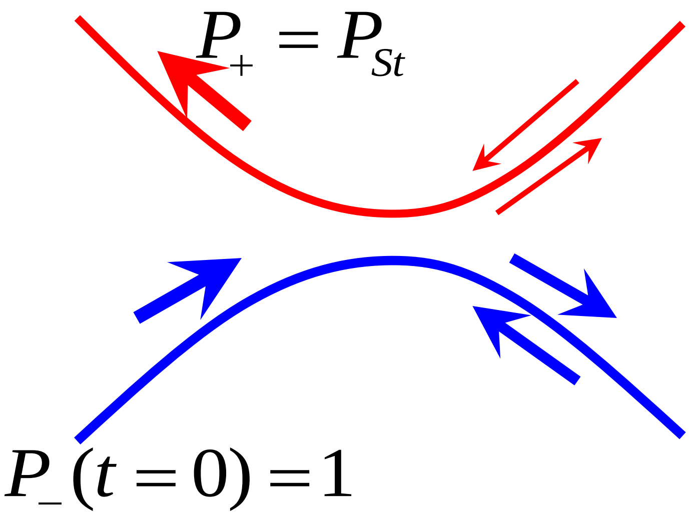
$$\boxed{P_{LZ} = \exp(-2\pi\delta)}$$

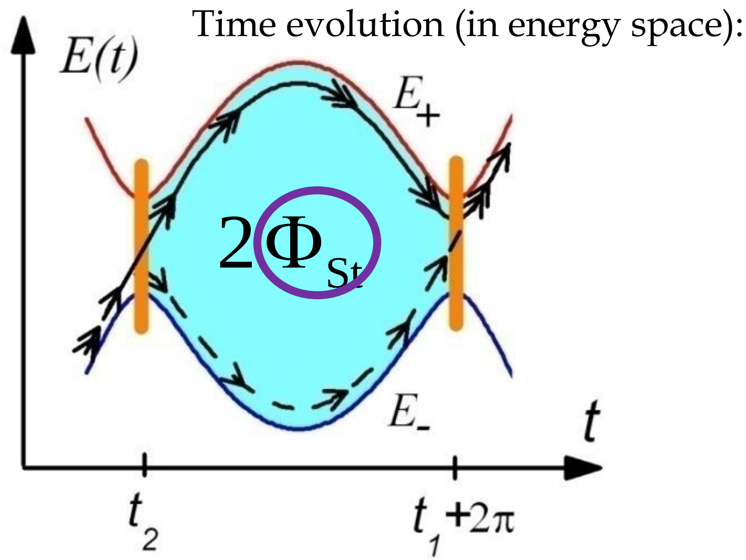
$$\delta = \frac{\Delta^2}{4A\hbar\omega}, \quad A = A\sqrt{1-\left(\frac{\varepsilon_0}{A}\right)^2}$$

In this single-passage case the Schrödinger equation can be solved analytically, and numerically.



Double passage: Stückelberg oscillations, like the Mach-Zehnder interferometer





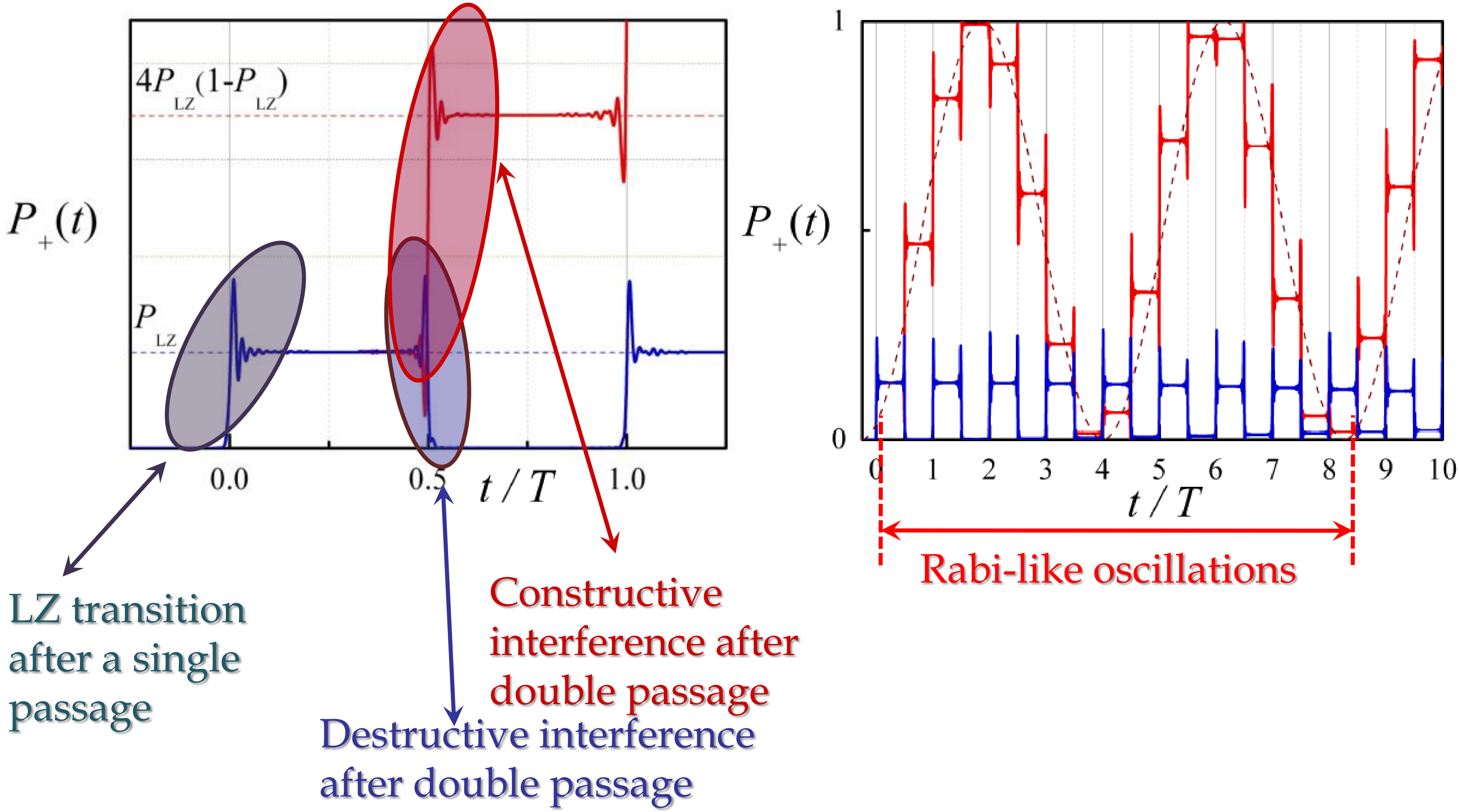
Final upper-level occupation probability: $\boxed{P_{St} = 4P_{LZ}(1-P_{LZ})\sin^2\Phi_{St},}$

where $\Phi_{St} \approx \frac{1}{2\hbar}\int_{t_2}^{t_1+2\pi} dt\sqrt{\varepsilon(t)^2+\Delta^2} = \frac{1}{2\hbar}\int_{t_2}^{t_1+2\pi} dt\left(E_+ - E_-\right).$

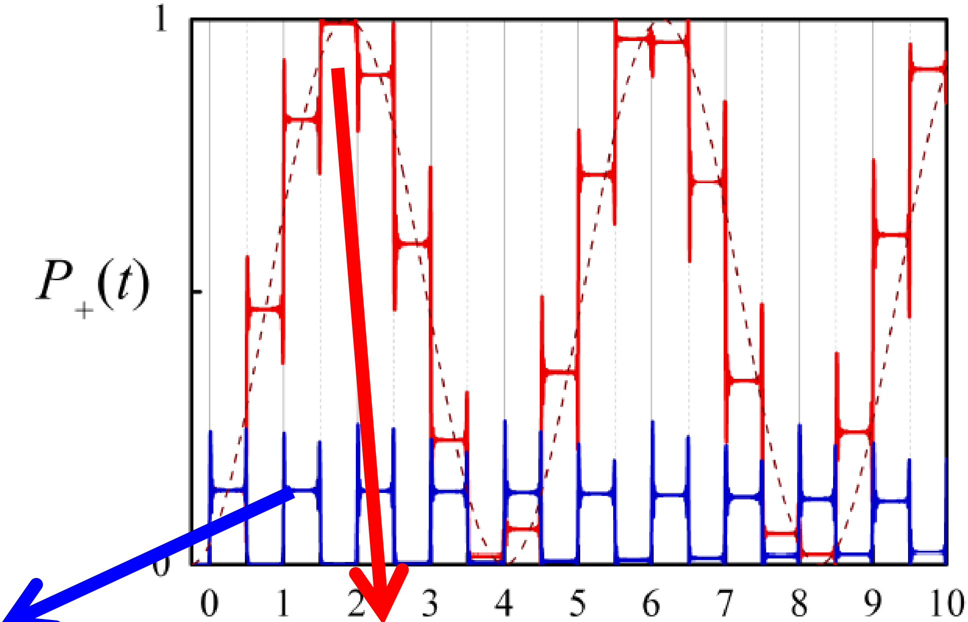
In most problems of microscopic physics, the phase averages out:

$$\bar{P}_{St} = 2P_{LZ}(1-P_{LZ}) = P_{LZ}\times(1-P_{LZ}) + (1-P_{LZ})\times P_{LZ}.$$

In contrast, for mesoscopic systems it matters. NB: $\boxed{\Phi_{St} = \Phi_{St}(\Delta, \varepsilon_0, A, \omega).}$

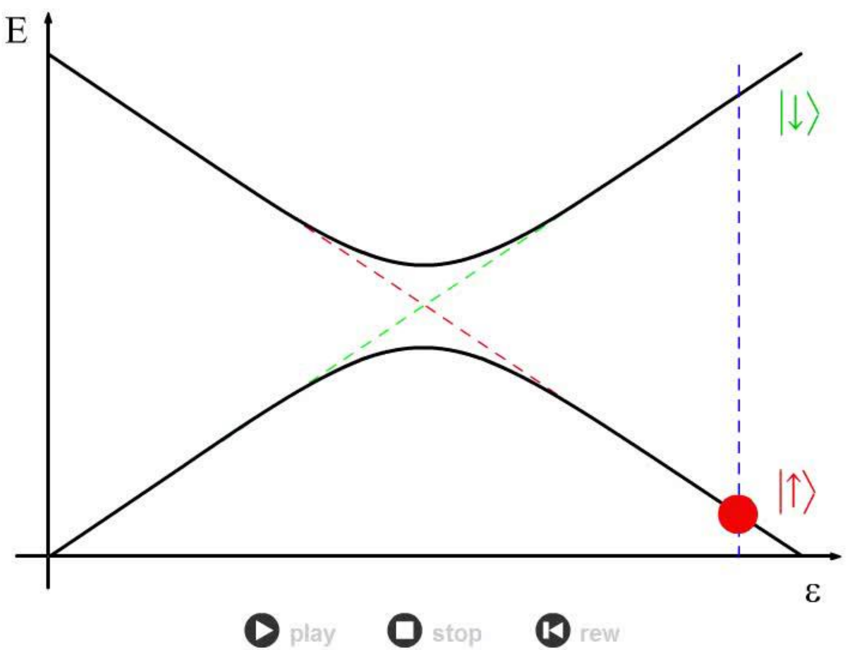


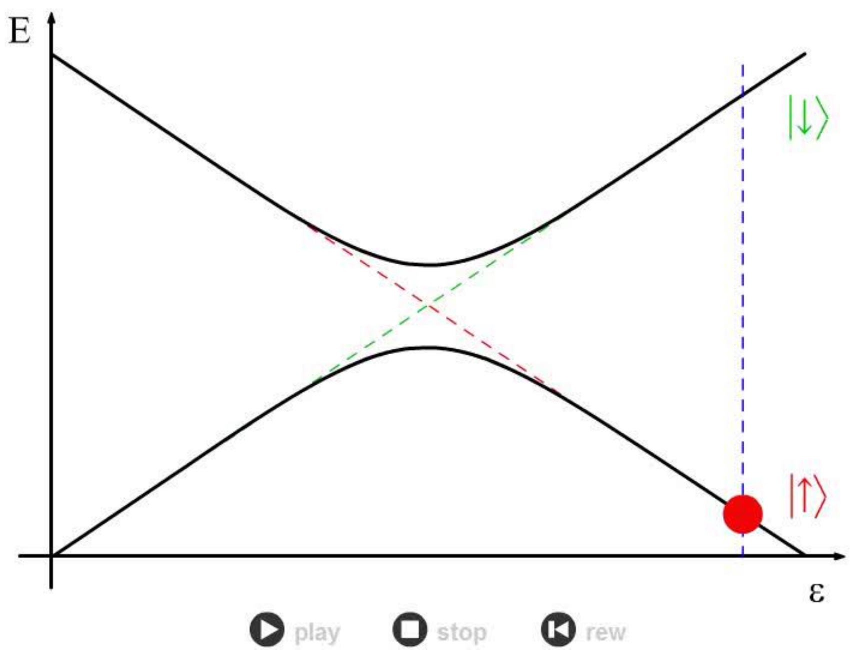
4P_LZ(1-P_LZ)
P_LZ
P_+(t)
0.0
0.5
1.0
t / T
LZ transition after a single passage
Constructive interference after double passage
Destructive interference after double passage
P_+(t)
1
0
0 1 2 3 4 5 6 7 8 9 10
t / T
Rabi-like oscillations



Destructive interference

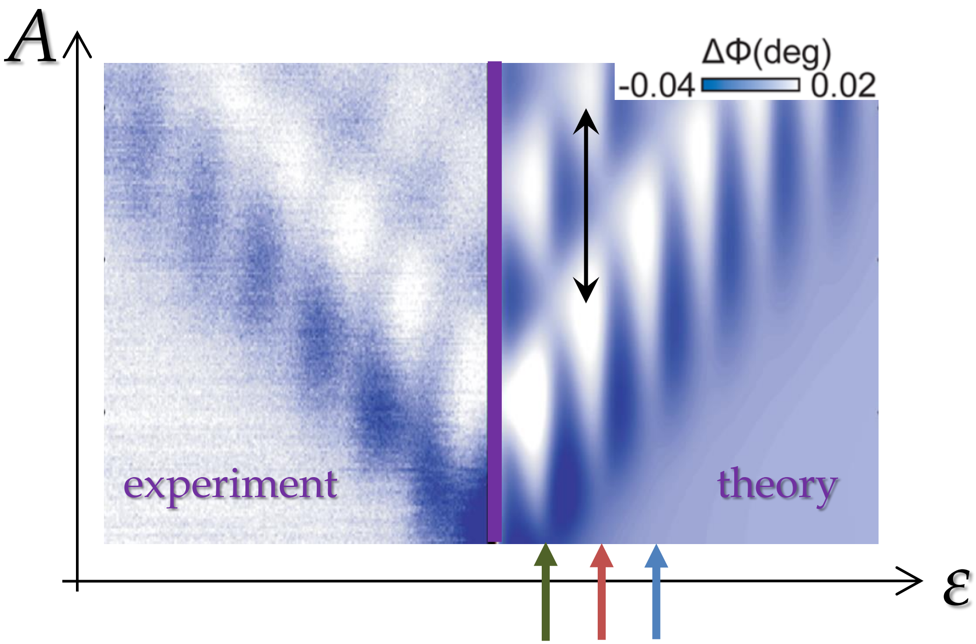
Constructive interference

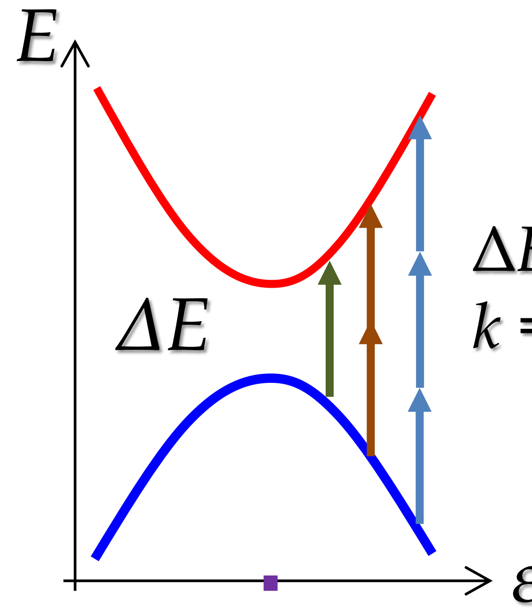




Landau-Zener-Stückelberg-Majorana interferometry

Phase shift ΔΦ in the rf resonator depends on the qubit state





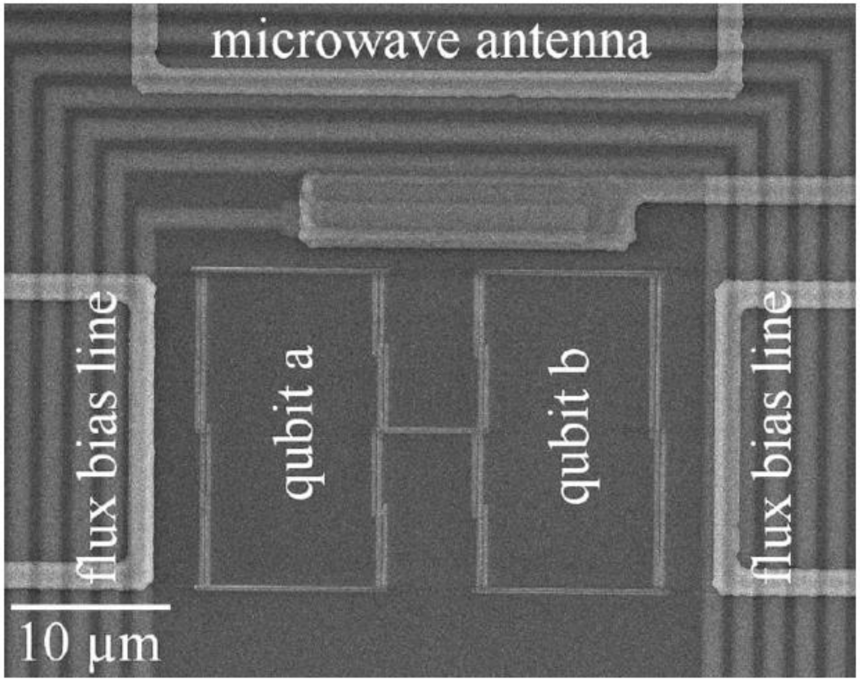
$\Delta E \approx k \cdot \hbar\,\omega$

$k = 1,2,3,\ldots$

Multiphoton resonances => parameters of qubits / spectroscopy
Stückelberg oscillations => power calibration
Resonances' shape => relaxation parameters

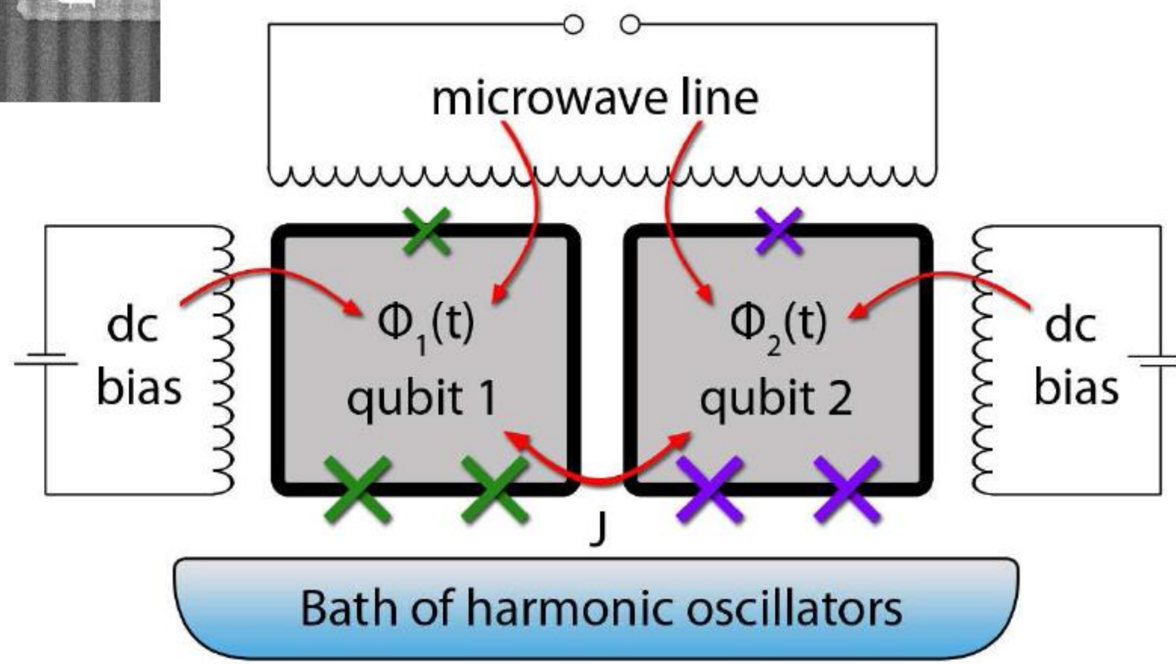
LZSM-interferometry allows the system to evolve from its ground state into any desirable superposition state, allowing control and manipulation of individual quantum systems.

Example: excitation of a two-qubit system

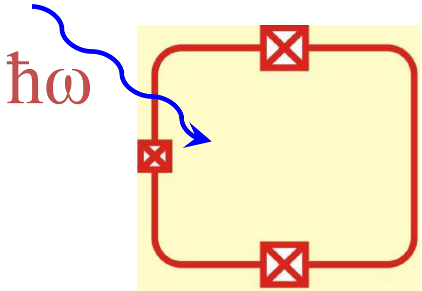


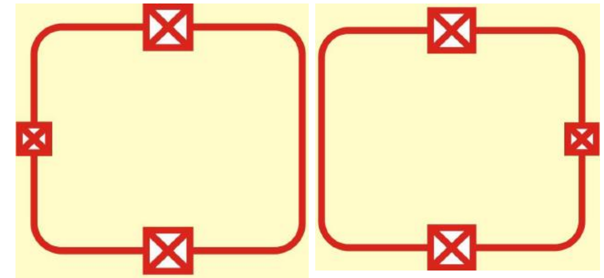
Systems of coupled superconducting qubits can be realized, probed and controlled.

Micrograph (top) and scheme (right)



Artificial atom and molecule

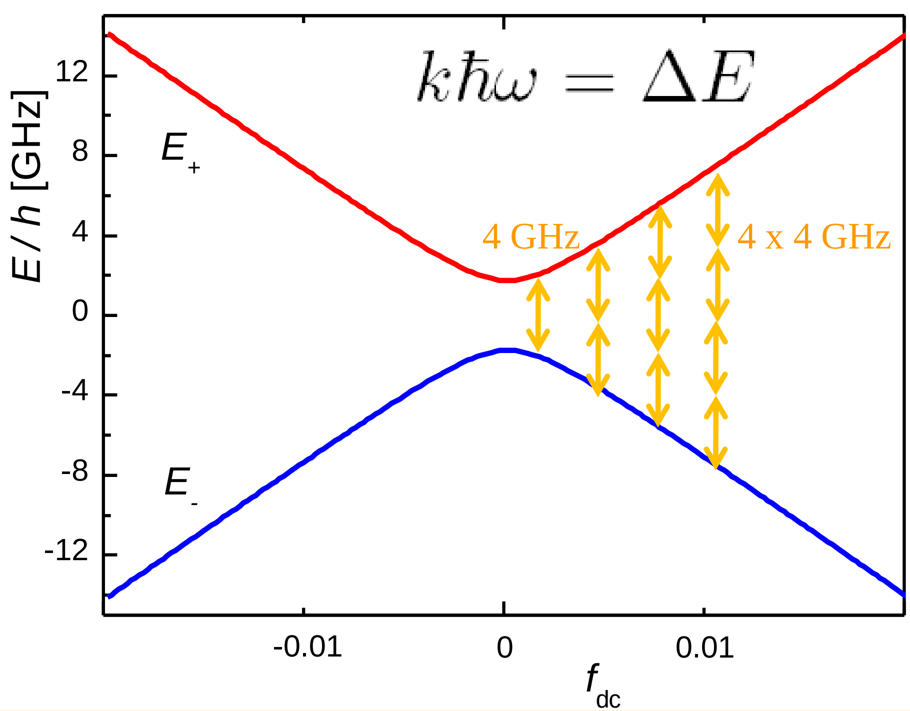


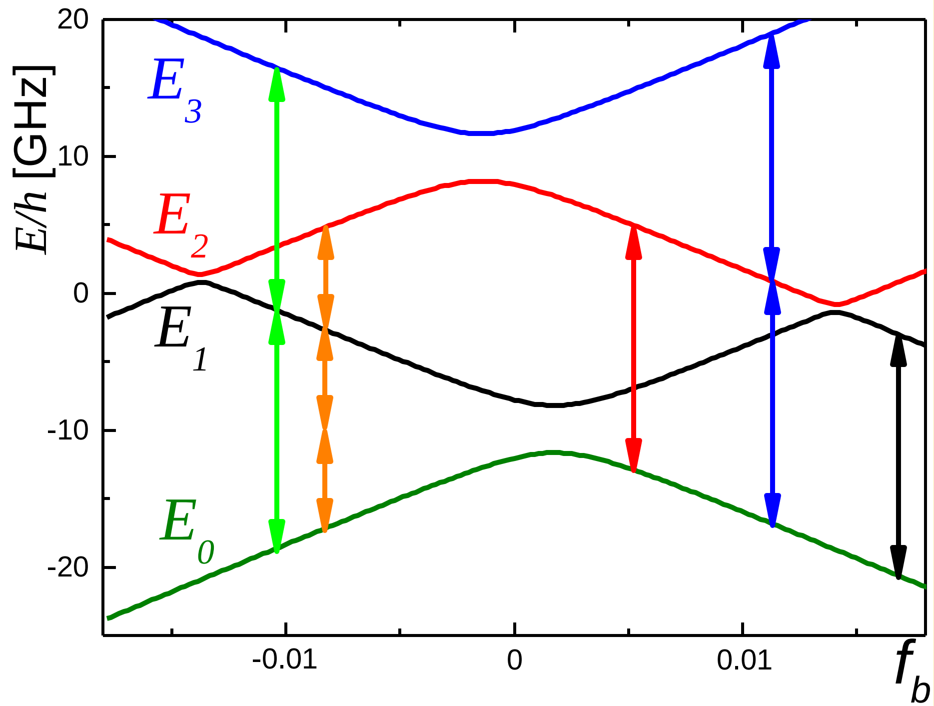


$$H = -\frac{\Delta}{2}\sigma_x - \frac{\varepsilon(t)}{2}\sigma_z \qquad \varepsilon(t) = \varepsilon_0 + A\sin\omega t,$$

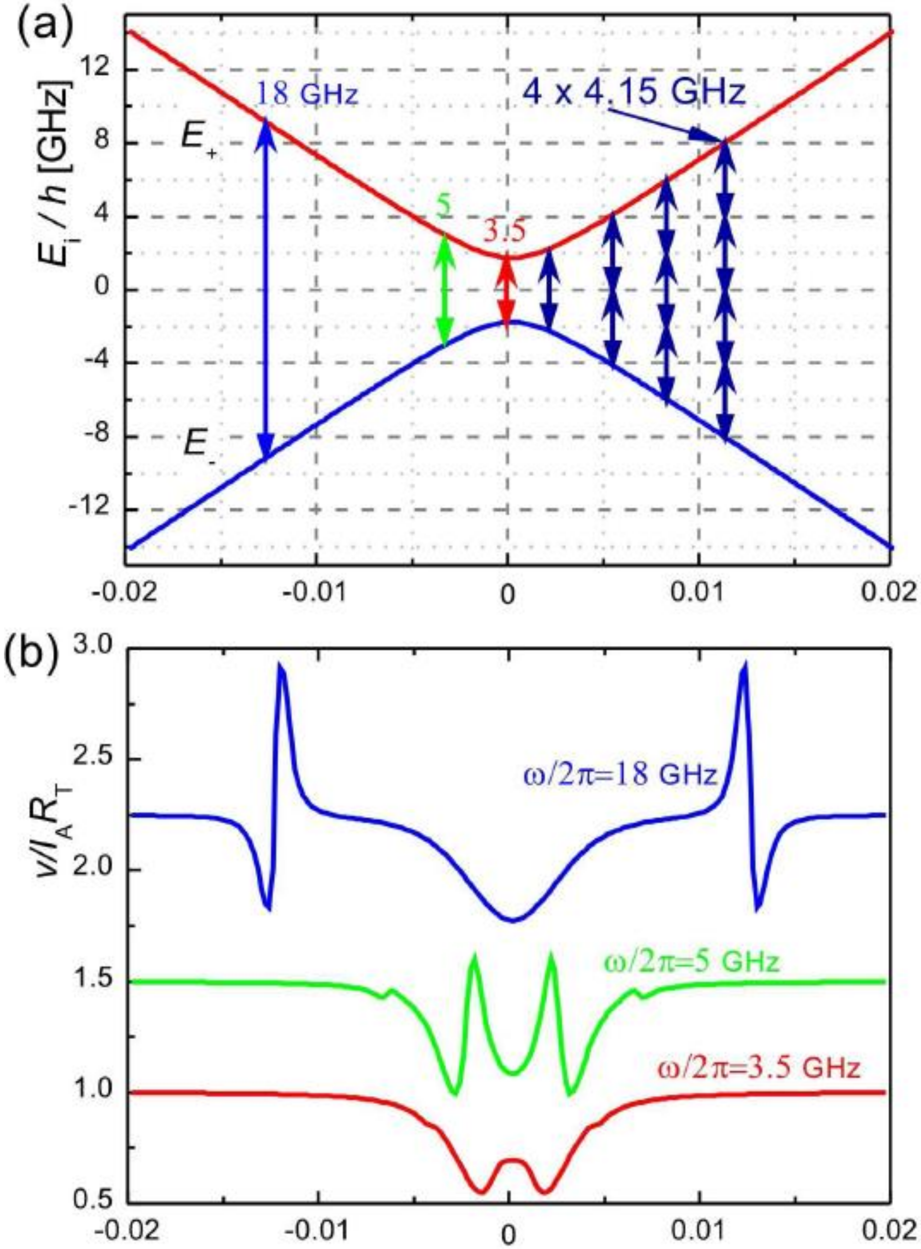
$$\Delta E = \sqrt{\Delta^2 + (I_{\mathrm{p}}\Phi_0 f_{\mathrm{dc}})^2}$$

$$H = \sum_{i=1}\left(-\frac{\Delta_i}{2}\sigma_x^{(i)} - \frac{\varepsilon_i(t)}{2}\sigma_z^{(i)}\right) + \sum_{i,j}\frac{J_{ij}}{2}\sigma_z^{(i)}\sigma_z^{(j)}$$





Spectroscopy of the flux qubit

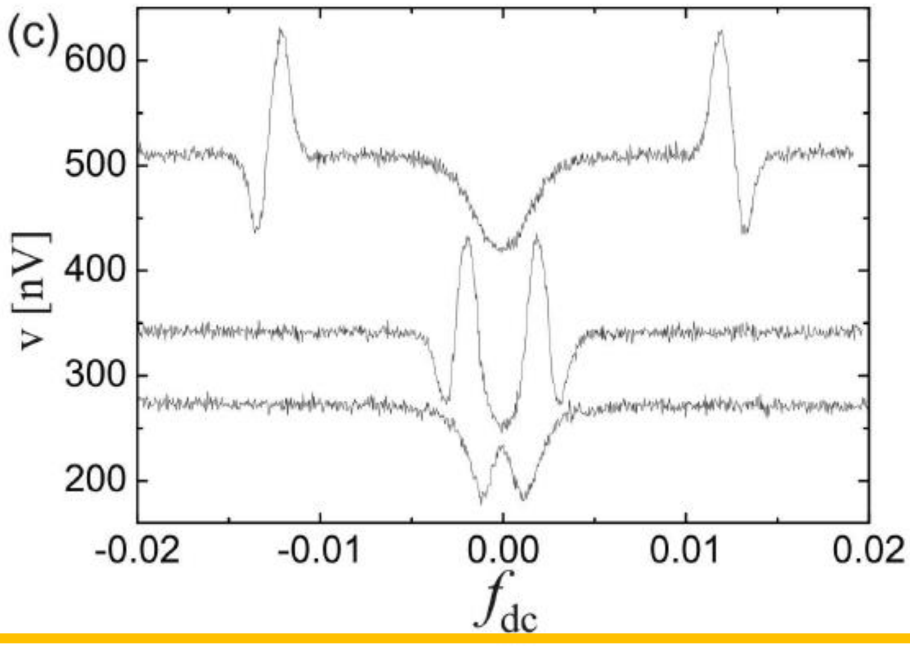


Qubit is excited at the resonant frequency: $\omega \approx \Delta E/\hbar$

Multiphoton resonances:

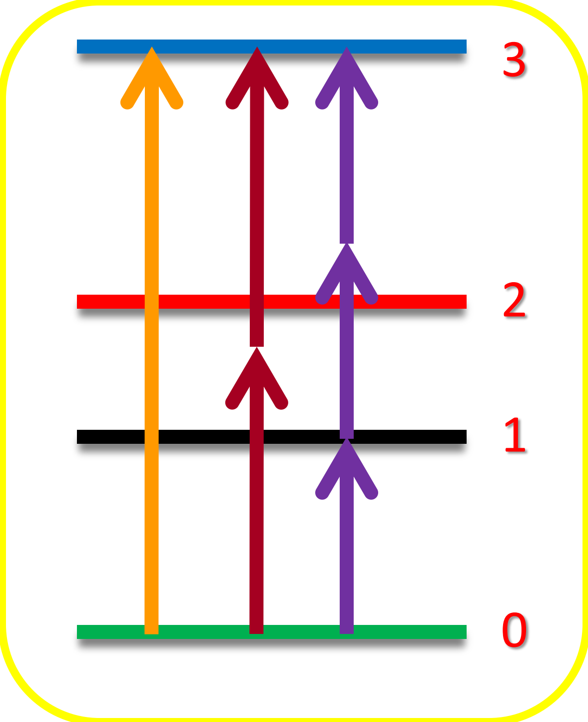
$$k\hbar\omega = \Delta E$$

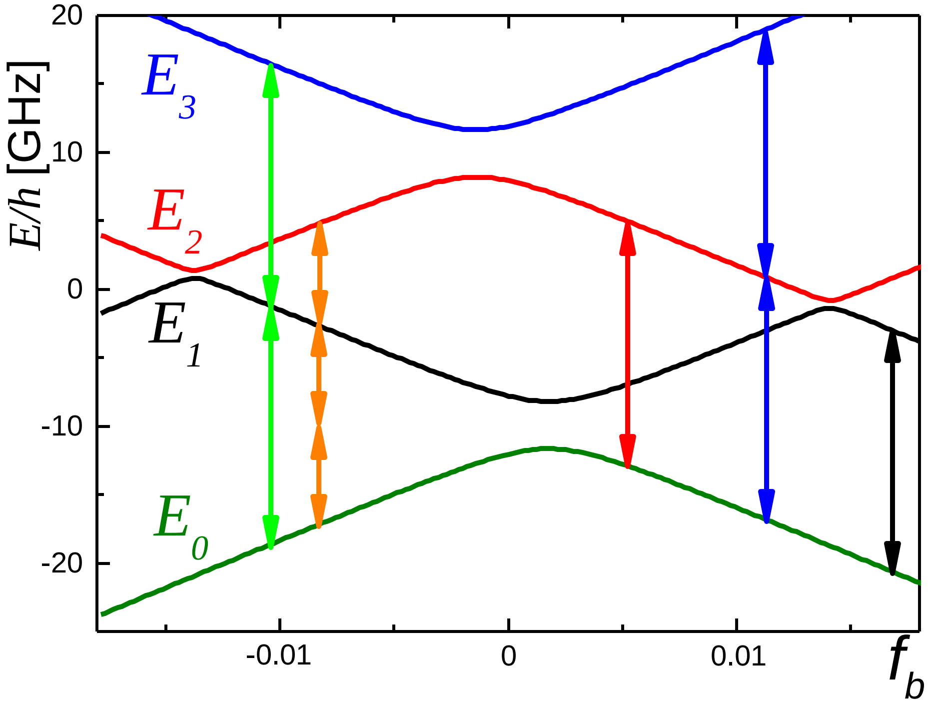
[PRL (2008) and PRB (2010)]



Multi-photon ecxitations in the two-qubit system

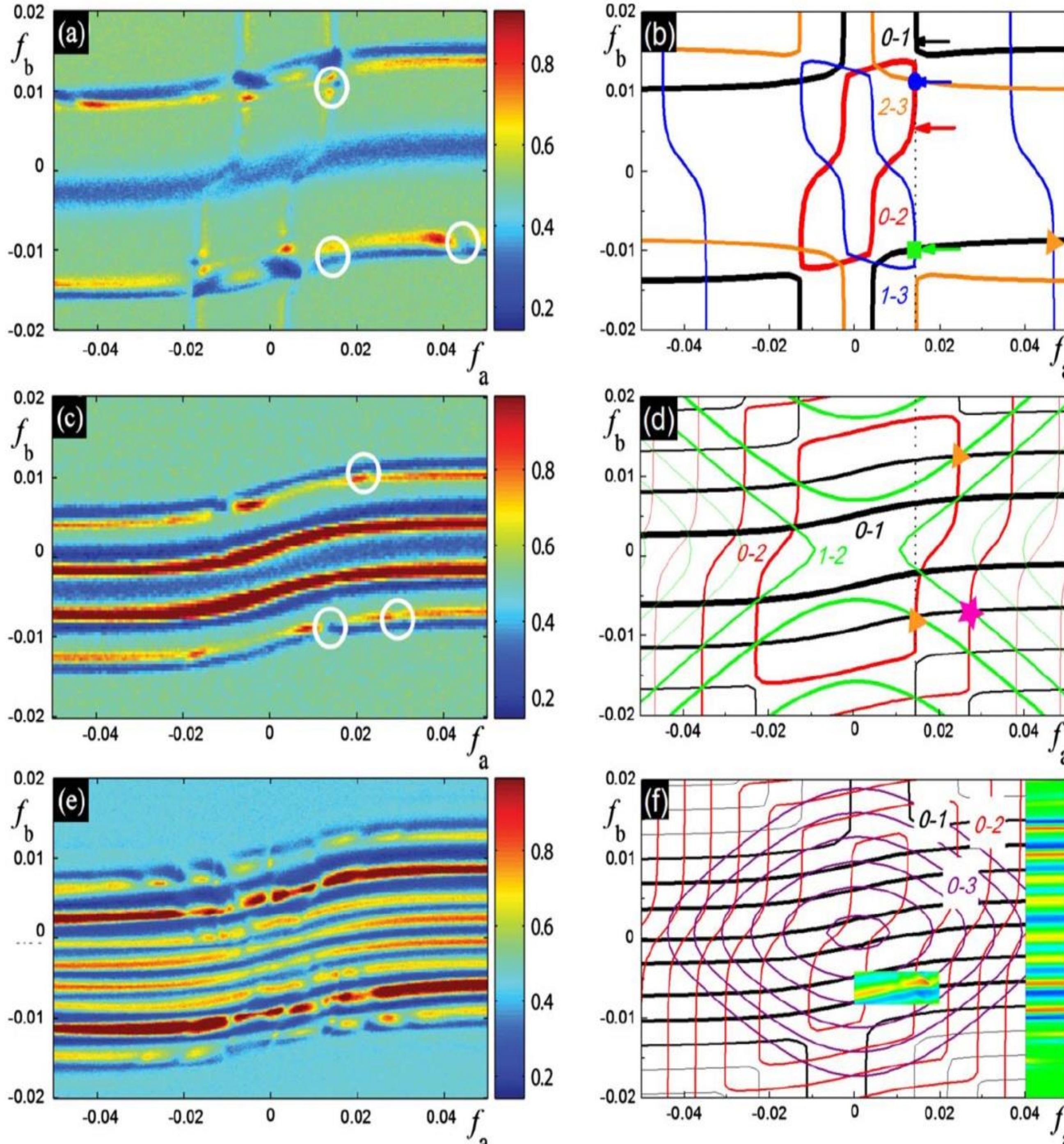
These can be direct and ladder-type transitions





Multi-photon ecxitations in the two-qubit system

Experimental results can be understood by comparing with the energy contour lines at $\Delta E_{ij}(f_a, f_b) = k \cdot \hbar\omega$

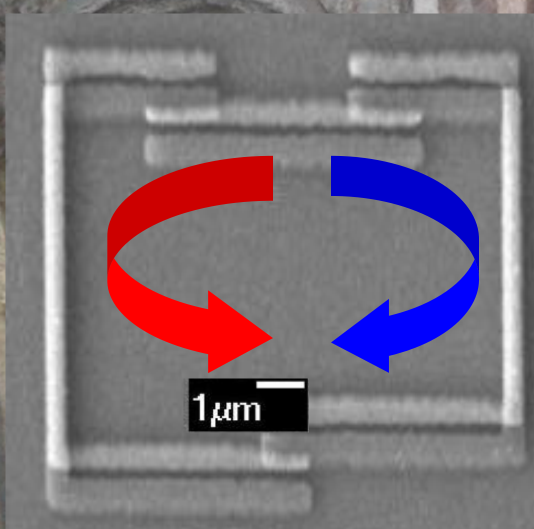


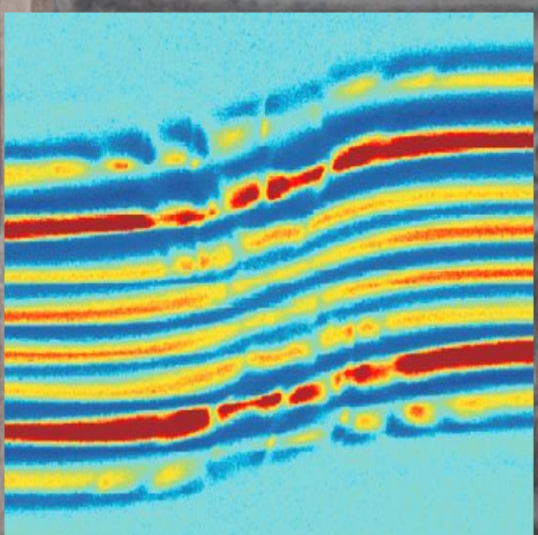
CONCLUSIONS

Superconducting circuits can behave as artificial atoms and molecules

They can be reliably created and controlled

In particular, they can experience multi-photon transitions





FOR FURTHER READING

Superconductivity: V. V. Schmidt, The Physics of Superconductors (Springer-Verlag, Berlin Heidelberg, 1997); (MCNMO, Moscow, 2000).

Quantum engineering: A. M. Zagoskin, Quantum Engineering: Theory and Design of Quantum Coherent Structures (CUP, Cambridge, 2011).

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Summer reading: R. Penrose, The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics (Oxford University Press, Oxford, 2002); (URSS, Moscow, 2003).

and

G. Greenstein and A. G. Zajonc, The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics (Jones and Bartlett Publishers, Sudbury, MA, 2006); (Intellect, Moscow, 2008).