Linear optics

Stefan Scheel, Universität Rostock
Summer School on Modern Quantum Technologies

Menu

Outline:

- linear optics and LOQC
- measurement-induced nonlinearities
- lossy devices

Linear-optical elements

linear optics: set of optical elements whose Hamiltonian is bilinear in the photonic creation and annihilation operators, $\hat{H} = \sum_{ij} c_{ij} \hat{a}_i^\dagger \hat{a}_j$

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} T & R \\ -R^* & T^* \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$

- unitary transformation SU(2) for lossless devices $|T|^2 + |R|^2 = 1$
- examples: beam splitters, phase shifters etc., but no squeezing ($\hat{H} \propto \hat{a}^2, \hat{a}^{\dagger 2}$)
- unitary evolution described by operator

$$\hat{U} = T^{\hat{n}_1} e^{-R^* \hat{a}_2^\dagger \hat{a}_1} e^{R \hat{a}_1^\dagger \hat{a}_2} T^{-\hat{n}_2}$$
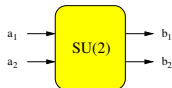
LOQC is a paradigm for universal quantum computing (KLM)!

E. Knill, R. Laflamme, and G.J. Milburn, Nature **409**, 46 (2001).

Quantum optics of macroscopic systems

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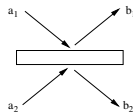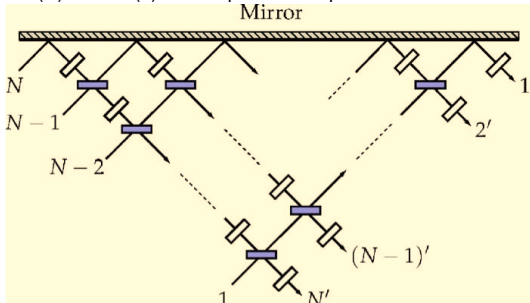LOQC is a paradigm for universal quantum computing (KLM)!

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Quantum optics of macroscopic systems

Linear-optical elements

generalization: N-port interferometer $\hat{b}_k \mapsto \sum_{j=1}^{N} U_{jk}\,\hat{a}_j$, $\hat{b}_k^\dagger \mapsto \sum_{j=1}^{N} U_{jk}^*\,\hat{a}_j^\dagger$

decomposition of U(N) into SU(2) beam splitters and phase shifters

M. Reck, A. Zeilinger, H.J. Bernstein, and P. Bertani, Phys. Rev. Lett. **73**, 58 (1994).

Linear-optical elements

qubits in linear optics:

- polarization (horizontal/vertical): $|0\rangle_L = |H\rangle, |1\rangle_L = |V\rangle$
- single photon in two spatial modes ('dual rail'): $|0\rangle_L = |1\rangle \otimes |0\rangle, |1\rangle_L = |0\rangle \otimes |1\rangle$
- qubits in Fock basis

single-qubit gates: realizable with U(2) operations, i.e. phase shifts and beam splitters

two-qubit gates: e.g. Bell state from logical qubits $|H, H\rangle \mapsto \frac{1}{\sqrt{2}}(|H, V\rangle + |V, H\rangle)$ not a linear operation on the level of operators!

$\Rightarrow$ need effective interaction of photons, not only mixing

Quantum optics of macroscopic systems

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Two-qubit gates

example: controlled-phase gate $|q_1, q_2\rangle \overset{CZ}{\mapsto} (-1)^{q_1 q_2}|q_1, q_2\rangle$

could be realized with cross-Kerr nonlinearity $\hat{H}_{\text{Kerr}} = \kappa \hat{n}_1 \hat{n}_2$

natural nonlinearities too small:

- vacuum QED photon-mixing amplitude $\propto \alpha^4$

- Kerr phase shift from nonlinear crystals on single photon level $\simeq 10^{-18}$ (other nonlinear effects also contribute and disturb cross-Kerr effect)

- electromagnetically-induced transparency (EIT) $\simeq 10^{-5}$

- possibly Rydberg-EIT

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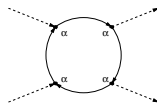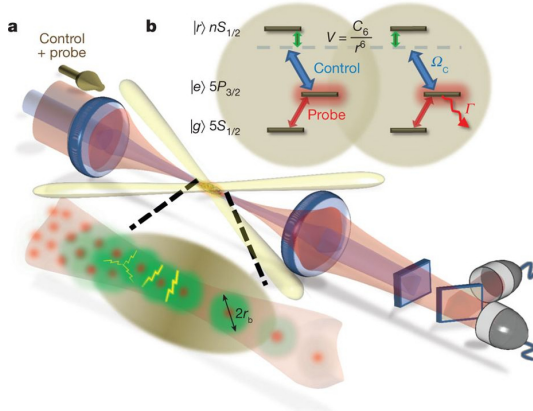- possibly Rydberg-EIT

Quantum optics of macroscopic systems

Rydberg-EIT

T. Peyronel, O. Firstenberg, Q. Liang, S. Hofferberth, A. Gorshkov, T. Pohl, M. Lukin, V. Vuletic, Nature **488**, 57 (2012).
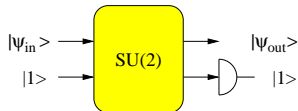
Measurement-induced nonlinearities

basis idea: use higher-dimensional (linear) transformations and select an outcome suitable for the desired purpose (post-selection)

example: single beam splitter

$$\hat{U} = T^{\hat{n}_1} e^{-R^* \hat{a}_2^\dagger \hat{a}_1} e^{R \hat{a}_1^\dagger \hat{a}_2} T^{-\hat{n}_2}$$
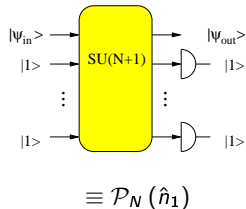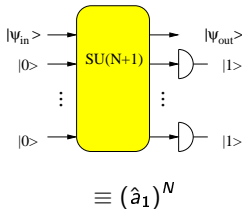

single beam splitter fed with a single photon; nonlinear operator conditioned on detecting a single photon at the output: $\hat{Y}_1 = \langle 1_2 | \hat{U} | 1_2 \rangle = T^{\hat{n}_1 - 1} \left[|R|^2 - |T|^2 \hat{n}_1 \right]$

conditional probability state-dependent: $p = || \hat{Y}_1 | \psi_{in} \rangle ||^2$

Quantum optics of macroscopic systems

Measurement-induced nonlinearities

generation of nonlinear operations using ancilla networks

$$\equiv \left(\hat{a}_1^\dagger \right)^N \qquad\qquad \equiv (\hat{a}_1)^N \qquad\qquad \equiv \mathcal{P}_N \left(\hat{n}_1 \right)$$
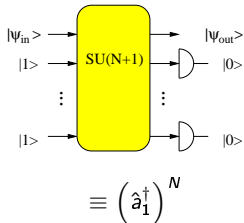
S. Scheel, K. Nemoto, W.J. Munro, and P.L. Knight, Phys. Rev. A **68**, 032310 (2003).

Measurement-induced nonlinearities

example: nonlinear sign shift gate

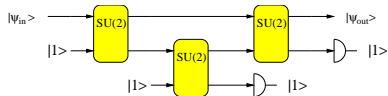$$|\psi_{\text{in}}\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle \mapsto |\psi_{\text{out}}\rangle = c_0|0\rangle + c_1|1\rangle - c_2|2\rangle$$

conditional operator: $e^{i\pi\hat{n}(\hat{n}-1)/2}$, but only $1 - \hat{n}(\hat{n}-1)$ needed

$\Rightarrow$ second-order polynomial in $\hat{n}$

$\Rightarrow$ two ancilla modes

Measurement-induced nonlinearities

example: controlled-phase gate
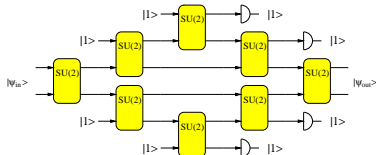
$$
\begin{aligned}
|00\rangle &\mapsto |00\rangle \\
|01\rangle &\mapsto |01\rangle \\
|10\rangle &\mapsto |10\rangle \\
|11\rangle &\mapsto e^{i\varphi}|11\rangle
\end{aligned}
$$

conditional operator: $\hat{C}_\varphi = 1 - \left(1 - e^{i\varphi}\right) \hat{n}_1 \hat{n}_2$

inside interferometer: $\hat{N}_1 \otimes \hat{N}_2,\ \hat{N}_i = 1 - \frac{1}{2}\left(1 - e^{i\varphi}\right) \hat{n}_i \left(\hat{n}_i - 1\right)$

$\Rightarrow$ second-order polynomial in $\hat{n}_i$

$\Rightarrow$ two ancilla modes in each arm

Quantum optics of macroscopic systems

Scaling of success probabilities

abstract network for single-mode gates $\Rightarrow$ upper bounds on success probabilities
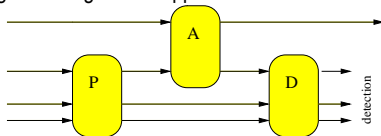
A: 'active' beam splitter, couples signal modes to ancillas
P: preparation stage for ancilla states
D: detection stage, transforms into detectable state

generalized nonlinear sign shift gate
$$|\psi_{\text{in}}\rangle = c_0|0\rangle + c_1|1\rangle + \ldots + c_N|N\rangle \mapsto |\psi_{\text{out}}\rangle = c_0|0\rangle + c_1|1\rangle + \ldots - c_N|N\rangle$$

under mild assumptions on ancilla states: $p_{\texttt{max}} = \dfrac{1}{N^2}$

S. Scheel and K.M.R. Audenaert, New J. Phys. **7**, 149 (2005).

Quantum optics of macroscopic systems

Quantum operations and decoherence

single-mode quantum mechanics describes state evolution in
terms of unitary operators

$\hat{\varrho}' = \hat{U}\hat{\varrho}\hat{U}^{\dagger}$, $\hat{U} = e^{i\hat{H}}$ ($\hat{H}$: Hamiltonian)

e.g. lossless (ideal) beam splitter

$\rho \longrightarrow$ | U | $\longrightarrow \rho'$

single-mode quantum mechanics generally is an idealization requiring perfectly isolated
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QIP requirement: manipulations as designed and well-controlled quantum operations
performed on quantum states

but: isolated systems cannot be (externally) manipulated
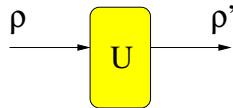
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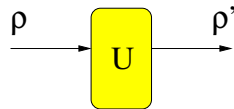but: isolated systems cannot be (externally) manipulated

Quantum optics of macroscopic systems

Quantum operations and decoherence

Generically, for a system to be (externally) manipulable, it has to be in contact with an environment

Kraus (completely positive, trace-preserving) map:

$$\hat{\varrho}' = \sum_i \hat{W}_i \hat{\varrho} \hat{W}_i^\dagger \,, \quad \sum_i \hat{W}_i^\dagger \hat{W}_i = \hat{I}$$

output fidelity: $\mathrm{Tr}[\hat{\varrho}_{\text{desired}} \hat{\varrho}'] < 1$

e.g. lossy (realistic) beam splitter

ρ U ρ'

Quantum optics of macroscopic systems

Quantum optics of the lossy beam splitter

idealised (lossless) beam splitter realizes an SU(2) transformation on the level of photonic amplitude operators

$$\hat{\varrho}' \left[\hat{\mathbf{a}}, \hat{\mathbf{a}}^\dagger \right] = \hat{\varrho} \left[\mathbf{T}^+ \hat{\mathbf{a}}, \mathbf{T}^\mathcal{T} \hat{\mathbf{a}}^\dagger \right]$$

with unitary transformation matrix $\mathbf{T} \in$ SU(2)

$$\mathbf{T} = \begin{pmatrix} T & R \\ -R^* & T^* \end{pmatrix}$$

energy (probability) conservation: $\mathbf{T}\mathbf{T}^+ = \mathbf{I} \equiv |T|^2 + |R|^2 = 1$

However: beam splitters are made of 'stuff', more precisely, linearly responding dielectrics

$\Rightarrow$ expect losses to occur, so that $|T|^2 + |R|^2 < 1$

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QED in dielectrics: add Langevin noise to Heisenberg's equations of motion

here: input-output relations for amplitude operators $\equiv$ discrete-time Heisenberg equations

$$\hat{\mathbf{b}} = \mathbf{T}\hat{\mathbf{a}} + \mathbf{A}\hat{\mathbf{g}}, \quad \hat{\mathbf{g}} : \text{device noise operators}$$

no longer unitary, $\mathbf{T}\mathbf{T}^+ \neq \mathbf{I}$, but instead $\mathbf{T}\mathbf{T}^+ + \mathbf{A}\mathbf{A}^+ = \mathbf{I}$

construct unitary transformation in larger space, $\hat{\boldsymbol{\alpha}} = (\hat{\mathbf{a}}, \hat{\mathbf{g}}), \hat{\boldsymbol{\beta}} = (\hat{\mathbf{b}}, \hat{\mathbf{h}})$

$$\hat{\boldsymbol{\beta}} = \boldsymbol{\Lambda}\hat{\boldsymbol{\alpha}} \quad \text{with} \quad \boldsymbol{\Lambda} = \begin{pmatrix} \mathbf{T} & \mathbf{A} \\ -\mathbf{S}\mathbf{C}^{-1}\mathbf{T} & \mathbf{C}\mathbf{S}^{-1}\mathbf{A} \end{pmatrix}, \quad \mathbf{C} = \sqrt{\mathbf{T}\mathbf{T}^+}, \mathbf{S} = \sqrt{\mathbf{A}\mathbf{A}^+}$$

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Quantum optics of the lossy beam splitter

lossy beam splitter is an open system!

$$\hat{\varrho}' \left[\hat{\mathbf{a}}, \hat{\mathbf{a}}^{\dagger} \right] = \mathrm{Tr}^{(D)} \, \hat{\varrho} \left[\Lambda^{+} \hat{\boldsymbol{\alpha}}, \Lambda^{T} \hat{\boldsymbol{\alpha}}^{\dagger} \right]$$

energy/probability/information flow into absorbing matter

example 1: N-photon Fock state:

$$|\psi_{\mathrm{in}}\rangle = |n, 0\rangle \mapsto \hat{\varrho}' = \sum_{k=1}^{n} \binom{n}{k} |T_1|^{2k} \left(1 - |T_1|^2 \right)^{n-k} |k\rangle\langle k|$$

example 2: coherent state: $|\alpha\rangle \mapsto |\mathbf{T}\alpha\rangle$

Quantum optics of macroscopic systems

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Example: PT-symmetric system with gain and loss

system with gain: $\mathbf{T}\mathbf{T}^{+} + \sigma\mathbf{A}\mathbf{A}^{+} = \mathbf{I}$

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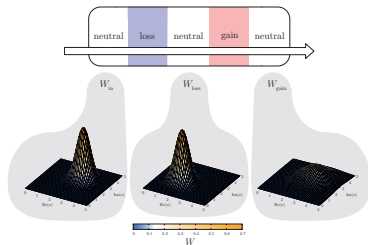
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  still a coherent state

- transmission through gain channel
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  displaced thermal state!

$\Rightarrow$ PT-symmetric systems with gain and loss do not exist in the quantum optical realm!

S. Scheel and A. Szameit, EPL **122**, 34001 (2018).

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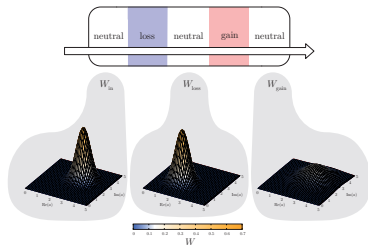
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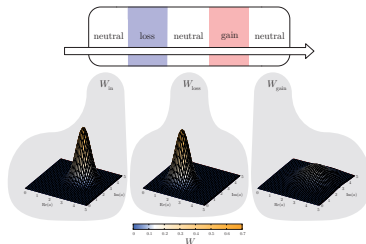
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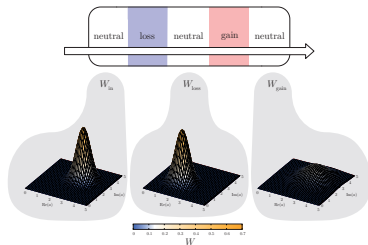
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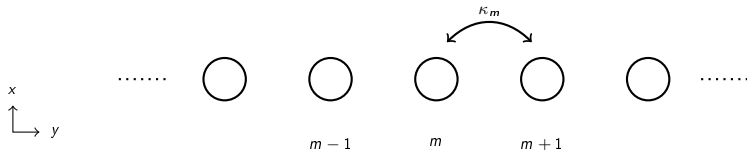# Quantum optics of macroscopic systems

Lossy coupled waveguides

for lossy coupled waveguides use Lie-Trotter formula: $e^{A+B} = \lim_{N \to \infty} (e^{A/N} e^{B/N})^N$

- Procedure in step j

  1. Couple W waveguide modes to device modes $\hat{a}^1, \ldots, \hat{a}^W \overset{\Lambda}{\longleftrightarrow} \hat{d}_j^1, \ldots, \hat{d}_j^W$

  2. Apply unitary U_j simulating waveguide system couplings over one step length

- Trace over device modes, perform limit $N \to \infty$
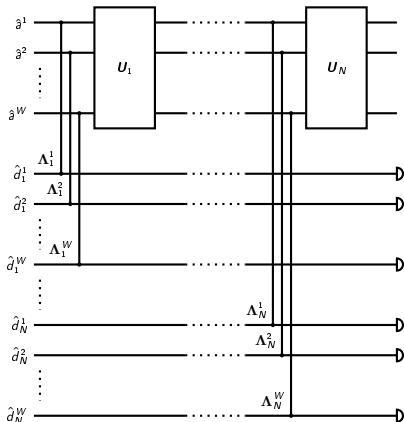
Quantum optics of macroscopic systems

1-D lossy coupled waveguide arrays

Hamiltonian & Lindblad master eq.

$$\hat{H} = \sum_{m=1}^{N} \sigma_m \hat{a}_m^\dagger \hat{a}_m + \sum_{m=1}^{N-1} \kappa_m \left(\hat{a}_m^\dagger \hat{a}_{m+1} + \hat{a}_{m+1}^\dagger \hat{a}_m \right)$$

$$\frac{\mathrm{d}}{\mathrm{d}z} \hat{\varrho} = \mathcal{L}\hat{\varrho} = -\mathrm{i}[\hat{H}, \hat{\varrho}] + \sum_{m=1}^{N} \gamma_m \left(\hat{a}_m \hat{\varrho}\, \hat{a}_m^\dagger - \frac{1}{2} (\hat{a}_m^\dagger \hat{a}_m \hat{\varrho} + \hat{\varrho}\, \hat{a}_m^\dagger \hat{a}_m) \right)$$

1-D lossy coupled waveguide arrays

transition to Liouville space by vectorization: $|n\rangle\langle m| \mapsto |n\rangle \otimes |m\rangle \equiv |n, m\rangle\rangle$

- $\frac{\mathrm{d}}{\mathrm{d}z}|\hat{\varrho}\rangle\rangle = \mathcal{L}|\hat{\varrho}\rangle\rangle$
- $\mathcal{L}$ generates quantum dynamical semigroup $\exp(\mathcal{L}z)$
- find eigendecomposition $\{|\psi_n\rangle\rangle, \langle\langle\psi_n|, \mu_n\}$ of Liouvillian $\mathcal{L}$

Lie algebra approach:

- find suitable Lie algebra basis $\{\hat{X}_i\}$ constructed from $\hat{a}, \hat{a}^\dagger$ that represents $\mathcal{L}$ and is complete under commutation
- calculate regular representation $\mathcal{R}(\mathcal{L})$ using $[\mathcal{L}, \hat{X}_i] = \mathcal{R}_{ij}\hat{X}_j$
- eigenvectors of $\mathcal{R}(\mathcal{L})$ are "superposition" ladder operators, eigenvalues μ_n the associated normalization constants
- apply ladder operators on vacuum state $|0\rangle\rangle$ to construct $\{|\psi_n\rangle\rangle, \langle\langle\psi_n|, \mu_n\}$
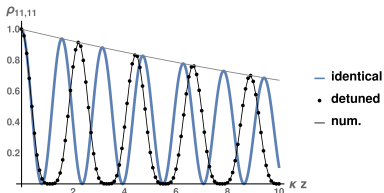
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$\Rightarrow$ exact decomposition $|\hat{\varrho}(z)\rangle\rangle = \mathrm{e}^{\mathcal{L}z}|\hat{\varrho}(0)\rangle\rangle = \sum_{n} \mathrm{e}^{\mu_n z}|\psi_n\rangle\rangle\langle\langle\psi_n|\hat{\varrho}(0)\rangle\rangle$

analytical solutions can be found in special cases

e.g. Hong-Ou-Mandel effect: identical waveguides, separable input $|1,1\rangle$

$\Rightarrow$ coincidence $\varrho_{11,11}(z) = \exp(-2\gamma z)\frac{1}{2}\left(1 + \cos(4\kappa z)\right)$

operator algebra: detuned waveguides,
non-separable input states
$\Rightarrow$ full eigendecomposition

Take-home messages

- linear optics $\equiv$ quantum optics of lossless beam splitters
- two-qubit gates require nonlinear interactions, generally by effective interactions
- measurement-induced nonlinearities provide all-optical tool to generate arbitrary effective operators
- probabilities scale polynomially with number of photons
- quantum optics of lossy beam splitters: open system approach
- suited for study of entanglement transport through complex photonic waveguide structures